Quiz 11

#8 p418 Evaluate $\int x^2 e^{x^3}\,dx$.

<u>Soln.</u> Let $u = x^3$. Then $du = 3x^2\,dx$. So $x^2\,dx = \frac{1}{3}\,du$.

Hence, $\int x^2 e^{x^3}\,dx = \frac{1}{3}\int e^u\,du = \frac{1}{3}e^u + C = \frac{1}{3}e^{x^3} + C$

#12 p418 Evaluate $\int \sec^2(2\theta)\,d\theta$.

<u>Soln.</u> Let $u = 2\theta$. Then $du = 2\,d\theta$. So, $d\theta = \frac{1}{2}\,du$.

$\int \sec^2(2\theta)\,d\theta = \frac{1}{2}\int \sec^2 u\,du = \frac{1}{2}\tan u + K = \frac{1}{2}\tan(2\theta) + K$

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28 p418 Evaluate $\int e^{\cos t}\sin t\,dt$.

<u>Soln.</u> Let $u = \cos t$. Then $du = -\sin t\,dt$. So, $\sin t\,dt = -du$.

Hence, $\int e^{\cos t}\sin t\,dt = -\int e^u\,du = -e^u + C = -e^{\cos t} + C$

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40 p.419 Evaluate $\int \frac{\sin x}{1+\cos^2 x}\,dx$.

<u>Soln.</u> Let $u = \cos x$. Then $du = -\sin x\,dx$. So $\sin x\,dx = -du$.

$\int \frac{\sin x}{1+\cos^2 x}\,dx = -\int \frac{du}{1+u^2} = -\tan^{-1} u + C = -\tan^{-1}(\cos x) + C$

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54 p.419 Evaluate $\int_0^1 (3t-1)^{50}\,dt$.

<u>Soln.</u> Let $u = 3t-1$. Then $du = 3\,dt$.
When $t=0$, $u=-1$. When $t=1$, $u=2$.

$$\int_0^1 (3t-1)^{50}\,dt = \frac{1}{3}\int_{-1}^{2} u^{50}\,du = \frac{1}{3(51)}\cdot u^{51}\Big|_{-1}^{2} = \frac{1}{153}\left(2^{51} - (-1)^{51}\right)$$

$$= \frac{2^{51}+1}{153}.$$

56 p419 Evaluate $\int_0^3 \frac{dx}{5x+1}$.

Soln 1 Without substitution:

$$\int_0^3 \frac{dx}{5x+1} = \frac{1}{5}\ln(5x+1)\Big|_0^3 = \frac{1}{5}\left(\ln 16 - \ln 1\right) = \frac{\ln 16}{5} = \frac{4\ln 2}{5}$$

Soln 2. With substitution:

Let $u = 5x+1$. $du = 5dx$. $x=0 \Rightarrow u=1$. $x=3 \Rightarrow u=16$.

$$\int_0^3 \frac{dx}{5x+1} = \frac{1}{5}\int_1^{16}\frac{du}{u} = \frac{1}{5}\left[\ln u\right]_1^{16} = \frac{1}{5}\ln 16 = \frac{4}{5}\ln 16$$

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62 p. 419 Evaluate $\int_0^{\pi/2} \cos x \sin(\sin x)\,dx$.

Soln. Let $u = \sin x$. Then $du = \cos x\,dx$. $x=0 \Rightarrow u=0$; $x=\frac{\pi}{2} \Rightarrow u=1$

Thus,

$$\int_0^{\pi/2} \cos x \sin(\sin x)\,dx = \int_0^1 \sin u\,du = -\cos u\Big|_0^1 = -\left[\cos 1 - \cos 0\right]$$

$$= 1-\cos 1$$

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72 p419 Evaluate $\int_0^{T/2} \sin\left(\frac{2\pi t}{T} - \alpha\right) dt$.

Soln. Let $u = \frac{2\pi t}{T} - \alpha$. Then $du = \frac{2\pi}{T}dt$. $\Rightarrow dt = \frac{T}{2\pi}du$

$t=0 \Rightarrow u=-\alpha$; $t=\frac{T}{2} \Rightarrow u = \frac{2\pi}{T}\left(\frac{T}{2}\right) - \alpha = \pi - \alpha$.

So,

$$\int_0^{T/2} \sin\left(\frac{2\pi}{T} - \alpha\right)dt = \frac{T}{2\pi}\int_{-\alpha}^{\pi-\alpha}\sin u\,du = \frac{T}{2\pi}\left[-\cos u\right]_{-\alpha}^{\pi-\alpha}$$

$$= -\frac{T}{2\pi}\left(\cos(\pi-\alpha) - \cos(-\alpha)\right)$$

$$= -\frac{T}{2\pi}\left(-\cos\alpha - \cos\alpha\right) \qquad \left(\begin{array}{l}\text{as } \cos(-\alpha) = \cos(\alpha) \\ \text{+ } \cos(\pi-\alpha) = -\cos\alpha\end{array}\right)$$

$$= \frac{T}{\pi}\cos\alpha.$$

82 p 420 — A bacteria population starts with 400 bacteria & grows at a rate of rate $= 450.268\, e^{1.12567t}$ bacteria per hour. How many bacteria will there be after 3 hours?

Soln. By the Net Change Theorem,

$$\Delta p = \int_0^3 r(t)\,dt$$

(See p. 406.)

where p denotes the population at time t. So

$$\Delta p = 450.268 \int_0^3 e^{1.12567t}\,dt = \frac{450.268}{1.12567}\, e^{1.12567t} \Big|_0^3$$

$$= \frac{450.268}{1.12576}\left[e^{1.12567(3)} - 1 \right]$$

$$\doteq 11,313.23 \implies \Delta p = 11,313 \text{ bacteria}$$

$$\therefore\ p = 400 + \Delta p = 11,713 \text{ bacteria}$$