

*No calculators. Show your work. Points will be deducted for inadequate explanations, using mathematical notation incorrectly, illegibility, not following directions, and not making obvious and conventional simplifications.*

1. Evaluate the following integrals using substitution and/or integration by parts. (40 pts.)

(a)  $\int \frac{\sin x}{\cos^2 x} dx.$

(b)  $\int e^{\sqrt{x}} dx.$

(c)  $\int x^3 e^{2x} dx.$

(d)  $\int x^{-2} \cos\left(1 + \frac{5}{x}\right) dx.$

(e)  $\int e^t \cos 3t dt.$

2. An animal population is increasing at a rate of  $29 + 37t$  animals per year where  $t$  is measured in years. By how much does the population increase between the end of the fourth year to the end of the tenth year?

3. Find the area of the region bounded by the curves  $y = x^2 - 8$  and  $y = 10 - x^2$ . (8 pts.)

4. Define the function  $S$  by

$$S(x) = \int_{4x}^2 \sin\left(\frac{\pi t^2}{2}\right) dt.$$

(a) Compute  $S(0.5)$ .

(2 pts.)

(b) Using the appropriate properties of integrals, write  $S(0) - S(1)$  as a single integral.

(2 pts.)

(c) Differentiate  $S(x)$  with respect to  $x$ .

(4 pts.)

5. Set up the integral for the volume of the solid obtained by rotating the region bounded by the given curve(s) about the specified line.

(a) bounded by the curve  $y = 4 - x^2$  and the  $x$ -axis and rotated about the  $x$ -axis. (Do not evaluate the integral.) (4 pts.)

(b) bounded by the curves  $y^2 = x$  and  $x = 2y$  and rotated about the  $y$ -axis. (Do not evaluate the integral.) (4 pts.)

6. Evaluate the definite integral:  $\int_0^2 (x-1)e^{(x-1)^2} dx$ . (8 pts.)

7. Find the volume of the solid obtained by rotating the region bounded by the curves  $y = x^2$  and  $x = y^2$  about the line  $y = 1$ .

**Extra Credit.** Evaluate the indefinite integral:  $\int \cot 2\theta d\theta$ . (4 pts.)