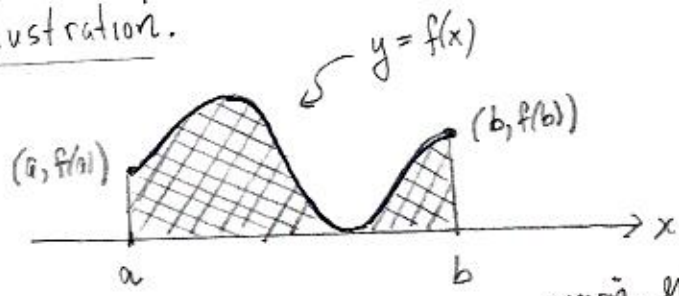


Definition of Area

Let f denote a function that is continuous and nonnegative on a closed interval $[a, b]$.

Illustration.



To approximate the area A of the region enclosed by the graph of $y = f(x)$, the x -axis, and from the vertical line $x=a$ to the vertical line $x=b$; execute the following steps:

- (i) Partition (divide) the interval $[a, b]$ into n subintervals of equal width $\Delta x = \frac{b-a}{n}$.
- (ii) Let $x_0 (=a), x_1, x_2, \dots, x_{n-1}, x_n (=b)$ denote the endpoints of these subintervals.
- (iii) From (i) and (ii) we have $x_i = a + i\Delta x$ for $i = 0, 1, 2, \dots, n$.

Then
$$A \approx f(x_1)\Delta x + f(x_2)\Delta x + \dots + f(x_n)\Delta x = \sum_{i=1}^n f(x_i)\Delta x.$$

This sum is called a Riemann sum using right endpoints and n subintervals. Let's denote it by R_n .

DEF. The area of the enclosed region is

$$A = \lim_{n \rightarrow \infty} R_n = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i)\Delta x.$$