

10-7 Graphing Linear Inequalities

Objective: To graph linear inequalities in two variables.

Vocabulary

Boundary A line that separates the coordinate plane into three sets of points: the points *on* the line, the points *above* the line, the points *below* the line.

If the boundary line is part of a graph, it is drawn as a *solid* line.

If the boundary line is *not* part of the graph, it is drawn as a *dashed* line.

Open half-plane Either of the two regions into which a boundary line separates the coordinate plane.

Closed half-plane The graph of an open half-plane and its boundary.

To graph a linear inequality in the variables x and y , when the coefficient of y is not zero:

1. **Transform** the given inequality into an equivalent inequality that has y alone as one side.
2. **Graph** the equation of the boundary. Use a solid line if the symbol \geq or \leq is used; use a dashed line if $>$ or $<$ is used.
3. **Shade** the appropriate region.

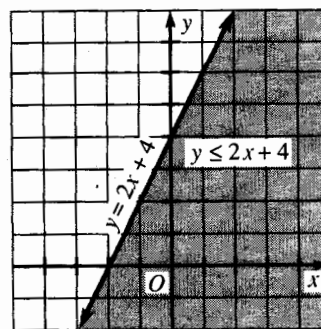
Example 1 Graph $2x - y \geq -4$.

Solution

1. Transform the inequality.

$$\begin{aligned} 2x - y &\geq -4 \\ -y &\geq -4 - 2x \\ y &\leq 4 + 2x \\ y &\leq 2x + 4 \end{aligned}$$

2. Draw the boundary line $y = 2x + 4$ as a *solid* line, since the symbol \leq includes the equals sign.
3. Shade the region *below* the line since the symbol \leq indicates the less than sign.



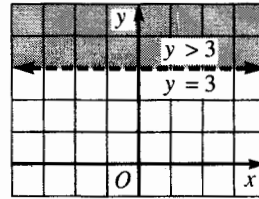
Check: Choose a point on the graph not on the boundary, say $(0, 0)$. See whether it satisfies the given inequality:

$$\begin{aligned} 2x - y &\geq -4 \\ 2(0) - 0 &\geq -4 \\ 0 &\geq -4 \quad \checkmark \end{aligned}$$

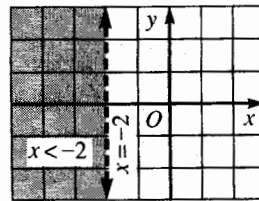
Thus, $(0, 0)$ is in the solution set, and the correct region has been shaded.

10-7 Graphing Linear Inequalities (continued)**Example 2** Graph $y > 3$.

Solution Graph $y = 3$ as a dashed horizontal line.
Any point above that line has a y -coordinate that satisfies $y > 3$.
Therefore, the graph of $y > 3$ is the open half-plane *above* the graph of $y = 3$.

**Example 3** Graph $x < -2$.

Solution Graph $x = -2$ as a dashed vertical line.
Any point to the left of that vertical line has an x -coordinate that satisfies $x < -2$.
Therefore, the graph of $x < -2$ is the open half-plane *to the left* of the graph of $x = -2$.

**Graph each inequality.**

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|----------------|------------------|--------------------|---------------------|
| 1. $y \geq 2$ | 2. $y > 2$ | 3. $x < 2$ | 4. $x \leq 2$ |
| 5. $x > 1$ | 6. $y < 3$ | 7. $y \leq -1$ | 8. $y \leq 3$ |
| 9. $y < x + 3$ | 10. $y > -x + 2$ | 11. $y \leq 4 - x$ | 12. $y \geq 1 - 2x$ |

Transform each inequality into an equivalent inequality with y as one side. Then graph the inequality.

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|-----------------------|----------------------------|--------------------------|
| 13. $x + y > 1$ | 14. $x - y \geq 2$ | 15. $x - 2y \leq -4$ |
| 16. $2x + y > -2$ | 17. $3x - y > 6$ | 18. $y - 2x \leq -6$ |
| 19. $2x - 3y \geq 6$ | 20. $2y - 3x < 0$ | 21. $3y - 5 > 2(x + 2y)$ |
| 22. $2y - 1 > 3x - 5$ | 23. $3(x - y) \geq 2x + 1$ | 24. $4y - 6 < 2(x + y)$ |

Mixed Review Exercises

Solve each system by whatever method you prefer.

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|----------------------------|-------------------------------|-------------------------------------|
| 1. $y = 2x$ $x - y = 1$ | 2. $m + n = 7$ $m - n = 3$ | 3. $8p + q = -6$ $8p - 6q = -20$ |
|----------------------------|-------------------------------|-------------------------------------|

Solve each open sentence and graph its solution set.

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|----------------------|------------------------|-----------------------------------|
| 4. $ 3p = 12$ | 5. $ 2p + 2 = 10$ | 6. $ 2x < 12$ |
| 7. $ 2x + 3 \geq 7$ | 8. $-5 \leq x + 1 < 4$ | 9. $2x + 3 > 5$ or $3 - x \geq 1$ |