Math 122 - Quiz 5 Name Key
February 1, 2019

No calculators. Show all necessary work.

1. State the hypotheses of the Mean Value Theorem. Then state the conclusion. (4 points.)

Hypotheses: Let a function f be continuous on $[a,b]$.
Let f be differentiable on (a,b).

Conclusion: Then there is a number c in (a,b) such that

$$f(b)-f(a) = f'(c)(b-a).$$

2. Find all numbers c that satisfy the conclusion of the Mean Value Theorem for $f(x) = \ln x$ on the interval $[1,4]$. (3 pts.)

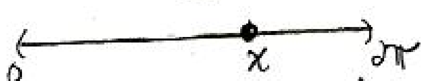
Note that $\ln x$ is continuous on $[1,4]$ and differentiable on $(1,4)$. (In fact it is differentiable on $(0,\infty)$ with $\frac{d}{dx}\ln x = \frac{1}{x}$.) So the hypotheses of the MVT are met. Thus there is a no. $c \in (1,4)$ s.t.

$$f(4)-f(1) = f'(c)(4-1) \Rightarrow \ln 4 - \ln 1 = \frac{1}{c}(3).$$

Since $\ln 1 = 0$ we have $\frac{3}{c} = \ln 4 \Rightarrow c = \frac{3}{\ln 4} = \frac{3}{2\ln 2}$.

3. Prove that $\sin x < x$ if $0 < x < 2\pi$. Hint. Use the Mean Value Theorem. (3 pts.)

Consider $f(x) = \sin x$ on $[0, 2\pi]$. The hypotheses of the MVT are satisfied b/c $\sin x$ is differentiable on $(-\infty, \infty)$; so in particular on $[0,2\pi]$. Let x be any point in the interval $(0, 2\pi]$.



Applying the MVT to $[0,x]$, we have

$$\frac{f(x)-f(0)}{x-0} = f'(c) \text{ for some } c \text{ in } (0,x).$$

$\Rightarrow \frac{\sin x}{x} = \cos c$. Since $\cos\theta < 1$ when $0 < \theta < 2\pi$, $\cos c < 1$.

Therefore $\frac{\sin x}{x} < 1 \Rightarrow \sin x < x$ for $0 < x \leq 2\pi$.

We proved more than was asked for.

Extra Credit. (1 pt.)
List any two antiderivatives of the function $f(x) = 3x^2$.

x^3, x^3+1 (any function of the form x^3+c where c = constant)