

1 Basic Concepts of Algebra

1-1 Real Numbers and Their Graphs

Objective: To graph real numbers on a number line, to compare numbers, and to find their absolute values.

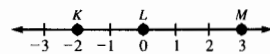
Vocabulary

Real numbers The set consisting of all the rational and irrational numbers.
 A rational number is the result of dividing an integer by a nonzero integer.
 An irrational number is one that is not rational.

Examples of rational numbers: 5 0 -2 6.3 $\frac{4}{7}$ 1.33...

Examples of irrational numbers: π $\sqrt{2}$ $\sqrt{5}$

Coordinate of a point The real number paired with that point on a number line. Example: The coordinate of point *M* is 3.



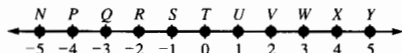
Origin The graph of zero on a number line.

Opposites On a number line, numbers that are the same distance from zero but on opposite sides of it. Example: -2 and 2

Absolute value of a number On a number line, the distance between the graph of the number and zero. Examples: The absolute value of 3 is 3 (write $|3| = 3$). The absolute value of -3 is also 3 (write $|-3| = 3$).

Symbols - (opposite of) | | (absolute value) > (is greater than) < (is less than)

Example 1 Find the coordinate of the point one fourth of the way from *S* to *W* on the number line below.



Solution The distance from *S* to *W* is 4 units. One fourth of 4 is 1. Move 1 unit to the right of *S* to find the desired point. The coordinate is 0.

- Find the coordinate of each point using the number line in Example 1. 8. -1 10. $\frac{3}{2}$
1. *Y* 5
 2. *T* 0
 3. *U* 1
 4. *P* -4
 5. *X* 4
 6. *N* -5
 7. The point halfway between *Q* and *Y* 1
 8. The point one fourth of the way from *R* to *V* 1
 9. The point two thirds of the way from *P* to *V* 0
 10. The point three fourths of the way from *Q* to *W* 1

Example 2 Write each statement using symbols.

- Five is greater than negative two.
- Negative ten is less than zero.

Solution a. $5 > -2$ b. $-10 < 0$

1-1 Real Numbers and Their Graphs (continued)

Write each statement using symbols.

14. $-1 > -6$
11. Four is less than six. $4 < 6$
12. Three is greater than negative three. $3 > -3$
13. Negative two is less than one. $-2 < 1$
14. Negative one is greater than negative six.
15. Negative one half is less than zero. $-\frac{1}{2} < 0$
16. Five is greater than negative eleven. $5 > -11$

Example 3 Graph the numbers 1.5 and -2.5 on a number line. Then write an inequality statement comparing them. Remember: the numbers increase from left to right.

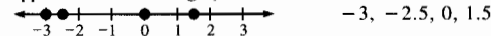
Solution $-2.5 < 1.5$, or $1.5 > -2.5$

Graph each pair of numbers on a separate number line. Graphs are given at the back of this Answer Key. Then write an inequality statement comparing the numbers.

17. -1 and 3 $-1 < 3$
18. 0 and -3 $0 > -3$
19. $\frac{1}{3}$ and $-\frac{1}{3}$ $\frac{1}{3} > -\frac{1}{3}$
20. $-\frac{2}{3}$ and $-\frac{4}{3}$ $-\frac{2}{3} > -\frac{4}{3}$
21. -1.25 and 0.75 $-1.25 < 0.75$
22. -0.75 and -1.5 $-0.75 > -1.5$

Example 4 Arrange -3, 0, 1.5, and -2.5 in order from least to greatest.

Solution First graph the numbers on a number line. Then write the numbers as they appear from left to right.



Arrange each list of numbers in order from least to greatest.

23. 4, -2, 0, -3, 2
24. -2.6, -3.2, -1.6, -2.2
25. $-\frac{4}{3}, -1, -\frac{2}{3}, \frac{1}{6}, \frac{1}{3}$
25. $-\frac{2}{3}, \frac{1}{6}, -1, \frac{1}{3}, -\frac{4}{3}$

Example 5 Find the value of each expression.

- $-|-14|$
- $|-8| - |-3|$

Solution First find the absolute values. Then simplify.

- $-|-14| = -(14) = -14$
- $|-8| - |-3| = 8 - 3 = 5$

Find the value of each expression.

26. $|-12|$ 12
27. $-|12|$ -12
28. $|4| - |-1|$ 3
29. $|-7| \cdot 3$ 21

Mixed Review Exercises

Tell whether each statement is true or false.

1. $2\frac{2}{3} - \frac{1}{2} > 2$ T
2. $\frac{5}{4} \div \frac{1}{2} = \frac{5}{2}$ T
3. $\frac{1}{4} + \frac{3}{5} = \frac{4}{9}$ F
4. $34.6 + 5.23 = 86.9$ F
5. $0.3 \times 0.2 < 0.6$ T
6. $10 \times 0.01 < 1$ T

NAME _____ DATE _____

1-2 Simplifying Expressions

Objective: To review the methods used to simplify numerical expressions and to evaluate algebraic expressions.

Vocabulary

Variable A symbol, usually a letter, used to represent one or more numbers.

Algebraic expression A numerical expression; a variable; or a sum, difference, product, or quotient that contains one or more variables.

Examples: $6 + 11$ x $x^2 - 5y + z$

Simplify To simplify an expression you replace it by the simplest or most common symbol having the same value.

Evaluate an expression To evaluate an algebraic expression, or find its value, replace each variable in the expression by a given value and simplify the result.

Power A product of equal factors. The repeated factor is the *base*. A positive *exponent* tells the number of times the base occurs as a factor. Example: $5 \times 5 \times 5 = 5^3$ is a power in which 5 is the base and 3 is the exponent.

Absolute value If x is positive or zero, $|x| = x$.
If x is negative, $|x| = -x$ (reads "the opposite of x ").

Symbols Grouping: () (parentheses) [] (brackets) — (fraction bar)

CAUTION Remember to use the correct order of operations. Expressions inside grouping symbols are simplified first, and then powers. Next, multiplication and division are done *in order from left to right*. Finally, addition and subtraction are done in order from left to right.

Example 1 Use one of the symbols $<$, $=$, or $>$ to make a true statement.

a. $5^2 + 3^2$? $(5 + 3)^2$ b. $(7 + 2) + 8$? $7 + (2 + 8)$

Solution Find the value of each side. Then compare the results.

a. $5^2 + 3^2 = 25 + 9 = 34$ b. $(7 + 2) + 8 = 9 + 8 = 17$
and $(5 + 3)^2 = 8^2 = 64$ and $7 + (2 + 8) = 7 + 10 = 17$,
Since 34 is less than 64, so $(7 + 2) + 8 = 7 + (2 + 8)$.
 $5^2 + 3^2 < (5 + 3)^2$.

Use one of the symbols $<$, $=$, or $>$ to make a true statement.

- $1 \cdot 4$? $1 \div 4$ >
- $4 \cdot 1$? $4 \div 1$ =
- $4^2 \cdot 5^2$? $(4 \cdot 5)^2$ =
- $4^2 + 5^2$? $(4 + 5)^2$ <
- $\frac{5+3}{5-3}$? $\frac{7+5}{7-5}$ <
- $\frac{5+4}{5-4}$? $\frac{8+4}{8-4}$ >
- $(8 + 5) + 1$? $8 + (5 + 1)$ =
- $(8 - 5) - 1$? $8 - (5 - 1)$ <
- $(8 \cdot 5) \cdot 2$? $8 \cdot (5 \cdot 2)$ =
- $(16 \div 4) \div 2$? $16 \div (4 \div 2)$ <

NAME _____ DATE _____

1-2 Simplifying Expressions (continued)

Example 2 Simplify $9^2 - 6 \div 3 + 4$.

$$\begin{aligned} \text{Solution} \quad 9^2 - 6 \div 3 + 4 &= 81 - \frac{6 \div 3}{3} + 4 \\ &= \frac{81 - 2}{79} + 4 \\ &= 83 \end{aligned}$$

Simplify the power: $9^2 = 9 \times 9$.
Divide.
Subtract.
Add.

Example 3 Simplify $11 + 5[8 - 3(6 - 4)]$.

$$\begin{aligned} \text{Solution} \quad 11 + 5[8 - 3(6 - 4)] &= 11 + 5 \cdot [8 - 3 \cdot 2] \\ &= 11 + 5 \cdot [8 - 6] \\ &= 11 + 5 \cdot 2 \\ &= 11 + 10 \\ &= 21 \end{aligned}$$

Subtract inside parentheses.
Multiply inside brackets.
Subtract inside brackets.
Multiply.
Add.

Simplify.

- a. $18 - 7 + 3 - 1$ **13** b. $18 - (7 + 3) - 1$ **7** c. $18 - (7 + 3 - 1)$ **9**
- a. $6 \cdot 4 + 5 \cdot 2$ **34** b. $6 \cdot (4 + 5) \cdot 2$ **108** c. $6 \cdot (4 + 5 \cdot 2)$ **84**
- a. $6^2 - 8 \div 2 + 5$ **37** b. $(6^2 - 8) \div 2 + 5$ **19** c. $(6^2 - 8) \div (2 + 5)$ **4**
- $3 \cdot 2^3 - (7^2 - 5^2)$ **0** 15. $16 - 3[9 - 2(5 - 3)]$ **1** 16. $[4(5 - 2) + 2^3] \div 2$ **10**
- $\frac{3^3}{4 - (4 - 1)}$ **27** 18. $\frac{1}{2} \left| \frac{1 + 9^2}{5^2} \right|$ **$\frac{41}{25}$** 19. $\frac{4^3 + 6}{4^2 - 6}$ **7**

Example 4 Evaluate each expression if $r = 2$, $s = 5$, $t = 6$, and $u = -9$.

a. $3r^2 + s - 6$ b. $\frac{t^2 - 3rs}{t + r^2}$ c. $3|u| - |s|$

Solution Substitute the given values for the variables. Then simplify.

a. $3r^2 + s - 6 = 3(2)^2 + 5 - 6 = 3 \cdot 4 + 5 - 6 = 12 + 5 - 6 = 11$
b. $\frac{t^2 - 3rs}{t + r^2} = \frac{6^2 - 3 \cdot 2 \cdot 5}{6 + 2^2} = \frac{36 - 30}{6 + 4} = \frac{6}{10} = \frac{3}{5}$
c. $3|u| - |s| = 3|-9| - |5| = 3 \cdot 9 - 5 = 27 - 5 = 22$

Evaluate each expression if $w = -8$, $x = 2$, $y = 3$, and $z = 7$.

- $y^2 - y + 1$ **7** 21. $3x^2 + x - 8$ **6** 22. $(xy - x)^3$ **64** 23. $\frac{2xy}{z^2 - y^3}$ **$\frac{6}{11}$**
- $\left(\frac{xyz}{x + y + z}\right)^2$ **$\frac{49}{4}$** 25. $|w| - |x|$ **6** 26. $|w| + 3|y|$ **17** 27. $|w|^2 - |z|^2$ **15**

NAME _____ DATE _____

1-4 Sums and Differences**Objective:** To review the rules for adding and subtracting real numbers.**Vocabulary****Rules for addition**

1. When adding two numbers with the *same sign*, you add the absolute values of the numbers and keep the sign.

$$\text{Example: } -6 + (-3) = -(|-6| + |-3|) = -(6 + 3) = -9$$

2. When adding two numbers with *opposite signs*, you subtract the *lesser* absolute value from the *greater* absolute value and keep the sign of the greater absolute value. Example: $4 + -9 = -(|-9| - |4|) = -(9 - 4) = -5$

Rule for subtraction To subtract a number, add its opposite: $a - b = a + (-b)$.

$$\text{Examples: } 4 - (-7) = 4 + 7 = 11 \quad -6 - 11 = -6 + (-11) = -17$$

Distributive property (of multiplication over subtraction)

$$\text{Example: } 4(5 - 2) = 4 \cdot 5 - 4 \cdot 2 = 12$$

Similar terms (or like terms) Terms with the same variables and exponents.

Examples: $5xy^2$ and $9xy^2$ are similar terms, but $6ab$ and $4ab^2$ are not.

Example 1 Simplify: a. $-13 + (-40)$ b. $-3.4 + 7.2$ c. $-14 - 28$

Solution

a. Use the *same sign* rule for addition. The answer will be negative.
 $-13 + (-40) = -(|-13| + |-40|) = -(13 + 40) = -53$

b. Use the *opposite signs* rule for addition. The answer will be positive since 7.2 has the greater absolute value.
 $-3.4 + 7.2 = |7.2| - |-3.4| = 7.2 - 3.4 = 3.8$

c. Use the rule for subtraction; add the opposite of 28.
 $-14 - 28 = -14 + (-28) = -(14 + 28) = -42$

Simplify.

1. $-52 + 17$ **-35** 2. $-27 - 14$ **-41** 3. $12 - (-33)$ **45** 4. $-16 + (-36)$ **-52**
 5. $-96 - (-28)$ **-68** 6. $0 - (-23.1)$ **23.1** 7. $-22.7 - (-22.7)$ **0** 8. $-16.5 - 12.5$ **-29**

Example 2 Simplify: a. $-17 + 15 - 19 + 31$ b. $(3 - 5) - (7 - 2)$

Solution

a. *Method 1:* Add left to right. *Method 2:* Group the negative terms and the positive terms.

$$\begin{array}{r} -17 + 15 - 19 + 31 \\ \underline{-2 - 19 + 31} \\ \underline{-21 + 31} \\ 10 \end{array} \qquad \begin{array}{r} -17 + 15 - 19 + 31 \\ \underline{(-17 - 19) + (15 + 31)} \\ \underline{-36 + 46} \\ 10 \end{array}$$

b. Be sure to perform the operations inside parentheses first.
 $(3 - 5) - (7 - 2) = -2 - 5 = -2 + (-5) = -7$

NAME _____ DATE _____

1-4 Sums and Differences (continued)**Simplify.**

9. $-18 - 19 + 12$ **-25** 10. $-2 + 8 - 6 + 3$ **3**
 11. $-12 + 3 + (-4) + 7$ **-6** 12. $1 - (3 - 4) + (7 - 8)$ **1**
 13. $3 - (5 + 1) - (1 - 4)$ **0** 14. $-6 - (12 - 20) + (6 - 10)$ **-2**
 15. $(-15 - 20) + [8 + (-3) - (-5)]$ **-25** 16. $[-5 + (-17)] - (3 - 7 + 2)$ **-20**
 17. $|2 - 6| + |8 - 20|$ **16** 18. $|-21 - 3| - |8 + (-12)|$ **20**

Example 3 Multiply: a. $3(7x - 2y)$
 b. $\frac{3}{4}(8x - 4y + 20)$

Solution Apply the distributive property.

$$\begin{aligned} \text{a. } 3(7x - 2y) &= 3(7x) - 3(2y) \\ &= 21x - 6y \\ \text{b. } \frac{3}{4}(8x - 4y + 20) &= \frac{3}{4}(8x) - \frac{3}{4}(4y) + \frac{3}{4}(20) \\ &= 6x - 3y + 15 \end{aligned}$$

Multiply.

19. $5(2x + 1)$ **$10x + 5$**
 20. $5(6 - y)$ **$30 - 5y$**
 21. $2(x - 8y)$ **$2x - 16y$**
 22. $\frac{2}{3}(6x - 3y)$ **$4x - 2y$**
 23. $\frac{1}{2}(10x - 4y - 2)$
 $5x - 2y - 1$
 24. $\frac{3}{5}(5x + 10y - 20)$
 $3x + 6y - 12$

Example 4 Simplify by combining similar terms.

a. $3(2x + 3y) - 5x$ b. $(2r + 5s - 3) + 3(r - 2s - 4)$

Solution

a. $3(2x + 3y) - 5x = 6x + 9y - 5x$ Use the distributive property.
 $= 9y + [6x - 5x]$ Group similar terms.
 $= 9y + (6 - 5)x$ Use the distributive property.
 $= 9y + x$ Simplify what is in parentheses.

b. $(2r + 5s - 3) + 3(r - 2s - 4)$
 $= 2r + 5s - 3 + 3r - 6s - 12$ Use the distributive property.
 $= [2r + 3r] + [5s - 6s] - 3 - 12$ Group similar terms.
 $= (2 + 3)r + (5 - 6)s - 15$ Use the distributive property.
 $= 5r - s - 15$ Simplify what is in parentheses.

Simplify by combining similar terms.

25. $19t + 3(8 + t)$ **$22t + 24$** 26. $2(3y - 5) - 18y$ **$-12y - 10$**
 27. $7(g - 2) + 8(g + 3)$ **$15g + 10$** 28. $3(9 - y) + 5(1 - y)$ **$32 - 8y$**
 29. $5d - 8 - 2d - 8$ **$3d - 16$** 30. $7x - 3y - 12x - (-5)y$ **$2y - 5x$**
 31. $(-6m - 3n + 2) + 5(m - 2n + 1)$
 $-m - 13n + 7$ 32. $4(3m - 6n) + 2(2m + 5n - 3)$
 $16m - 14n - 6$

1-5 Products

Objective: To review the rules for multiplying real numbers.

Vocabulary

Rules for multiplication

- The product of two real numbers with *like signs* is a *positive* real number.
Examples: $(3)(8) = 24$ $(-6)(-9) = 54$
- The product of two reals with *opposite signs* is a *negative* real number.
Example: $(5)(-14) = -70$
- A product of nonzero numbers is *positive* if the number of negative factors is *even*.
Example: $(-8)(-3)(-5)(-4) = 480$ (4 negative factors)
- A product of nonzero numbers is *negative* if the number of negative factors is *odd*.
Example: $(-6)(5)(-1)(-9) = -270$ (3 negative factors)
- The absolute value of the product of two or more numbers is the product of their absolute values. Example: $|(3)(-8)(4)| = |3| \cdot |-8| \cdot |4| = 96$

Multiplicative property of 0 The product of any number and zero is zero.

Multiplicative property of -1 The product of any number and negative one is the opposite of that number. Examples: $(5)(-1) = -5$ $(-1)(-18) = 18$

Property of the opposite of a product For all real numbers a and b , $-ab = (-a)(b) = (a)(-b)$.
Example: $-(3)(5) = (-3)(5) = (3)(-5) = -15$

Property of the opposite of a sum For all real numbers a and b , $-(a + b) = (-a) + (-b)$.
Example: $-[3 + (-19)] = -3 + 19 = 16$

Example 1 Simplify.

a. $(\frac{1}{2})(-8)(-6)(-\frac{1}{6})$ b. $(4x)(-3y)(2)$ c. $(5 - 7)(-8 + 3)$

Solution a. Multiply, beginning with the reciprocals. b. Reorder and regroup factors. c. Simplify expressions in parentheses first.

$(\frac{1}{2})(-8)(-6)(-\frac{1}{6})$ $(4x)(-3y)(2)$ $(5 - 7)(-8 + 3)$
 $= (\frac{1}{2})(-8)(1)$ $= 4(-3)(2)xy$ $= (-2)(-5)$
 $= (-4)(1)$ $= -24xy$ $= 10$
 $= -4$

Simplify.

- | | | |
|--|---------------------------------------|---|
| 1. $4(-2)(-3)(-5)$ -120 | 2. $(1.4)(-3)(-0.2)$ 0.84 | 3. $(-0.6)(-4)(-3)(-5.2)$ 37.44 |
| 4. $\frac{1}{2}(-6)(-\frac{1}{12})(-12)$ -3 | 5. $(4)(-\frac{3}{8})(12)$ -18 | 6. $(-\frac{1}{2})(-\frac{1}{3})(0)(-2)(-3)$ 0 |
| 7. $5(2x)(-3y)$ -30xy | 8. $(-\frac{1}{3})(3u)(-v)$ uv | 9. $(4x)(-2y)(-3z)$ 24xyz |
| 10. $(-a)(-b)(-c)$ -abc | 11. $(4 - 5)(3 + 8)$ -11 | 12. $(-12 - 3)(2 + 5)$ -105 |

1-5 Products (continued)

Example 2 Simplify: a. $(-2)^3(-\frac{1}{4})$ b. $3(-6) + 3(-2)$

Solution a. Write $(-2)^3$ as a product of factors. Then multiply. b. Follow the order of operations.

$(-2)^3(-\frac{1}{4})$
 $= (-2)(-2)(-2)(-\frac{1}{4})$
 $= (4)(-2)(-\frac{1}{4})$
 $= (-8)(-\frac{1}{4}) = 2$

$3(-6) + 3(-2) = -18 + (-6)$
 $= -24$

Or, apply the distributive property.
 $3(-6) + 3(-2) = 3[-6 + (-2)]$
 $= 3(-8)$
 $= -24$

Simplify.

- | | | |
|-----------------------------------|--|--------------------------------------|
| 13. $(14)(-1)^8(-3)^2$ 126 | 14. $(-3)^2(-\frac{1}{9})^2$ $\frac{1}{9}$ | 15. $(-2)^5(-3)(-1)$ -96 |
| 16. $6(-8) + 6(5)$ -18 | 17. $7(8.5) + 7(-1.5)$ 49 | 18. $(-4)^3(-1 - 1)(-3)$ -384 |

Example 3 Simplify $3(x^2 - 4) - 2(5x^2 - x)$.

Solution Distribute and simplify. Remember that a minus sign before a set of parentheses reverses each sign inside the parentheses.

$3(x^2 - 4) - 2(5x^2 - x) = 3x^2 - 12 - 10x^2 + 2x$
 $= -7x^2 + 2x - 12$

Simplify. $-2x^2 + 6x + 4$

$8x - 12y + 4z$

$-12a - 3$

- | | | |
|--|---|--|
| 19. $-2(x^2 - 3x - 2)$ | 20. $-4(-2x + 3y - z)$ | 21. $2(-6a - \frac{3}{2})$ 4a + 2b |
| 22. $(-r)(6s) + (3r)(-4s)$ -18rs | 23. $-5r - 4(2 + r)$ -9r - 8 | 24. $5(2a + b) - 3(2a + b)$ |
| 25. $2(p^2 + 2) - 3(2p^2 - p)$ -4p^2 + 3p + 4 | 26. $4(2x - 3y) - 2(-2y + 3x)$ 2x - 8y | 27. $m(n + 1) - 4(mn + 2)$ -3mn + m - 8 |

Mixed Review Exercises

Evaluate each expression if $a = -2$ and $b = 3$.

- | | | |
|--------------------------|----------------------------|------------------------|
| 1. $2a + b + 1$ 0 | 2. $- a + b $ 1 | 3. $a + b^2$ 7 |
| 4. $- a - b $ -5 | 5. $a - 2(b - 5)$ 2 | 6. $a - 4b$ -14 |

Name the property illustrated in each statement.

- | | | |
|--|---|--|
| 7. $8 \cdot 4 = 4 \cdot 8$ Comm. prop. of mult. | 8. $5 \cdot 1 = 5$ Identity prop. of mult. | 9. $\frac{2}{3} \cdot \frac{3}{2} = 1$ Prop. of reciprocals |
| 10. $-3 + 3 = 0$ Prop. of opposites | 11. $3 \cdot 6 + 3 \cdot 9 = 3(6 + 9)$ Dist. prop. | 12. $(6 + 2) + 8 = 6 + (2 + 8)$ Assoc. prop. of add. |

1-6 Quotients

Objective: To review rules for dividing real numbers.

Vocabulary

Division To divide by any *nonzero* real number, multiply by its reciprocal.

Examples: $15 \div 3 = 15 \cdot \frac{1}{3} = 5$ $12 \div \left(-\frac{4}{3}\right) = 12 \cdot \left(-\frac{3}{4}\right) = -9$

Since zero has no reciprocal, *division by zero is undefined.*

Rules for division

- The quotient of two real numbers with *like signs* is a *positive* real number.
Examples: $18 \div 3 = 6$ $(-22) \div (-2) = 11$
- The quotient of two real numbers with *opposite signs* is a *negative* real number.
Example: $6 \div (-3) = -2$
- For all *nonzero* real numbers a , b , and c :

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c} \quad \text{and} \quad \frac{a-b}{c} = \frac{a}{c} - \frac{b}{c}$$

Example 1 Simplify $-6 \div 9 \div \frac{1}{3}$. **Solution** Work from left to right.

$$\begin{aligned} -6 \div 9 \div \frac{1}{3} &= \left(-6 \cdot \frac{1}{9}\right) \div \frac{1}{3} \\ &= -\frac{2}{3} \div \frac{1}{3} \\ &= -\frac{2}{3} \cdot 3 = -2 \end{aligned}$$

Simplify.

1. $-63 \div (-9)$ 7 2. $-4 \div 16$ $-\frac{1}{4}$ 3. $-48 \div 8 \div (-2)$ 3
4. $-18 \div \frac{2}{3}$ -27 5. $-\frac{1}{3} \div \left(-\frac{1}{6}\right) \div (-4)$ $-\frac{1}{2}$ 6. $(-2)^3 \div [4(-6)]$ $\frac{1}{3}$

Example 2 Simplify: a. $\frac{(-6)(-8) \div (-2)}{4(-3)}$ b. $\frac{3\left(\frac{3}{4} - \frac{1}{4}\right)}{-\frac{1}{2} \div \frac{3}{2}}$

Solution Simplify the numerator and denominator separately. Then divide.

a. $\frac{(-6)(-8) \div (-2)}{4(-3)} = \frac{48 \div (-2)}{-12} = \frac{-24}{-12} = 2$

b. $\frac{3\left(\frac{3}{4} - \frac{1}{4}\right)}{-\frac{1}{2} \div \frac{3}{2}} = \frac{3\left(\frac{2}{4}\right)}{-\frac{1}{2} \cdot \frac{2}{3}} = \frac{\frac{3}{2}}{-\frac{1}{3}} = \frac{3}{2} \cdot (-3) = -\frac{9}{2}$

1-6 Quotients (continued)

Simplify.

7. $\frac{(-2)(-8)(-9)}{(-12)(-3)}$ -4 8. $\frac{(-4)(-12) \div (-3)}{8(-2)}$ 1 9. $\frac{7(-12) \div 14}{4(-3)}$ $\frac{1}{2}$
10. $\frac{42 - 32}{4 - (-3)}$ 1 11. $\frac{-8\left(-\frac{1}{2} - \frac{1}{4}\right)}{-\frac{3}{4} \div 3}$ -24 12. $\frac{-27\left[12 \div \left(-\frac{3}{4}\right)\right]}{12\left(-\frac{3}{4}\right)}$ -48

Example 3 Simplify $\frac{72 - 8x^2}{-4}$. **Solution** Apply the third rule for division on page 11. Then simplify.

$$\begin{aligned} \frac{72 - 8x^2}{-4} &= \frac{72}{-4} - \frac{8x^2}{-4} \\ &= -18 - (-2x^2) \\ &= -18 + 2x^2 \end{aligned}$$

Simplify.

13. $\frac{6x^2 - 21}{-3}$ $-2x^2 + 7$ 14. $\frac{56 - 4x^2}{-4}$ $x^2 - 14$ 15. $\frac{2 - (-x)^2}{-1}$ $x^2 - 2$
16. $\frac{-3x^2 - 32}{-3}$ $x^2 + 3$ 17. $\frac{12 + 4x - 8x^2}{4}$ $-2x^2 + x + 3$ 18. $\frac{-20x^3 - 15x^2 - 10x}{-5}$ $4x^3 + 3x^2 + 2x$

Example 4 Evaluate $\frac{(x^2 + 2)(x - 3)}{x - 1}$ for $x = -2$.

Solution Substitute -2 for x . Then simplify using the order of operations.

$$\begin{aligned} \frac{(x^2 + 2)(x - 3)}{x - 1} &= \frac{[(-2)^2 + 2](-2 - 3)}{-2 - 1} \\ &= \frac{(4 + 2)(-5)}{-3} \\ &= \frac{6(-5)}{-3} \\ &= \frac{-30}{-3} = 10 \end{aligned}$$

Evaluate each expression for the given values of the variables.

19. $\frac{x(x + 5)}{x - 2}$ a. $x = 1$ -6 b. $x = 0$ 0 c. $x = 4$ 18
20. $\frac{(y^2 - 2)(y + 3)}{y + 1}$ a. $y = -2$ -2 b. $y = 1$ -2 c. $y = 3$ $\frac{21}{2}$
21. $\frac{(n - 3)(n + 2)(n - 1)}{\frac{1}{4}n - 4}$ a. $n = -4$ 14 b. $n = 0$ $-\frac{3}{2}$ c. $n = 4$ -6
22. $\frac{a(a + 5)(a - 2)}{(a + 1)(a - 3)}$ a. $a = 1$ $\frac{3}{2}$ b. $a = -3$ $\frac{5}{2}$ c. $a = \frac{1}{2}$ $\frac{11}{10}$

1-7 Solving Equations in One Variable

Objective: To solve certain equations in one variable.

Vocabulary

Open sentence An equation or inequality that contains one or more variables.

Solution set The set of all values of a variable that make an open sentence true.

A solution is also called a *root*. A solution is said to *satisfy* an equation.

Transformations Changes that produce equivalent equations (ones with the same solution set). They include:

1. Simplifying either side of an equation.
2. Adding the same number to each side of an equation, or subtracting the same number from each side of an equation.
3. Multiplying (or dividing) each side of an equation by the same *nonzero* number.

Solve an equation Transform the equation into a simpler equivalent one whose solution set is easily seen.

Identity An equation that is satisfied by all values of the variable. The solution set of an identity is the set of all real numbers.

Symbols $\stackrel{?}{=}$ (Are they equal?) \therefore (therefore)
 \neq (is not equal to) \emptyset (empty or null set, the set with no members)

Example 1	Solve $3(2x - 3) = 4x + 7$.	(The goal is to get x alone on one side.)
Solution	$6x - 9 = 4x + 7$	Simplify the left side.
	$6x - 9 + 9 = 4x + 7 + 9$	Add 9 to each side.
	$6x = 4x + 16$	
	$6x - 4x = 4x + 16 - 4x$	Subtract $4x$ from each side.
	$2x = 16$	
	$\frac{2x}{2} = \frac{16}{2}$	Divide each side by 2.
	$x = 8$	
	Check: $3[2(8) - 3] \stackrel{?}{=} 4(8) + 7$	Substitute 8 for x in the given equation.
	$3(16 - 3) \stackrel{?}{=} 32 + 7$	Simplify each side.
	$3(13) \stackrel{?}{=} 39$	
	$39 = 39 \quad \checkmark$	
	\therefore the solution set is $\{8\}$.	

Solve. Check your work.

1. $4x - 6 = 2$ {2} 2. $6 = 3x + 3$ {1} 3. $\frac{1}{2}x - 4 = -2$ {4} 4. $7 - \frac{1}{5}y = -2$ {45}
5. $48 - 6x = 2x$ {6} 6. $x + 2 = 3x - 6$ {4} 7. $4(x - 3) = 2x - 6$ {3} 8. $3(1 - y) = 3y$ {1/2}

1-7 Solving Equations in One Variable (continued)

Example 2 Solve.

- a. $3x - (x - 5) = 2(x + 4)$ b. $3x - (x - 8) = 2(x + 4)$

Solution a. $3x - x + 5 = 2x + 8$ b. $3x - x + 8 = 2x + 8$
 $2x + 5 = 2x + 8$ $2x + 8 = 2x + 8$
 $5 = 8$ False!

No value of x will make this statement true.
 \therefore the solution set is \emptyset .

This is an identity. It is true for all values of x .
 \therefore the solution set is the set of all real numbers.

9. {2} 12. {-1}

Solve. Check your work when there is a single solution.

9. $5(x - 2) - 3 = -(x + 1)$ 10. $2x - 1 = 2(x + 4)$ \emptyset 11. $2(x + 2) = 2x + 4$ {real numbers} {12}
12. $4a + 1 = a - 5 - 3a$ 13. $1.2(u - 2) = 4.8$ {6} 14. $0.4(2r + 3) = 0.6r + 3.6$
15. $3(x - 2) = 5(x - 2)$ {2} 16. $\frac{1}{5}(x + 3) = x - 5$ {7} 17. $5z - (6 - z) = 4(1 - z)$ {1}
18. $11x - 3(4x - 2) = 2(8 + 2x)$ {-2} 19. $\frac{5k - 3(k - 2)}{4} = -6$ {-15}

Example 3 Solve the equation $T = c + ct$ for the variable t .

Solution

$$T = c + ct$$

$$T - c = c + ct - c$$

$$T - c = ct$$

$$\frac{T - c}{c} = \frac{ct}{c}$$

$$t = \frac{T - c}{c}$$

Work toward getting t alone on one side.
 Subtract c from each side.

Divide each side by c . (Assume $c \neq 0$.)

Solve each equation for the given variable.

20. $C = \pi d$ for d $d = \frac{C}{\pi}$ 21. $I = prt$ for r $r = \frac{I}{pt}$ 22. $3x - 4y = 8$ for y $y = \frac{3}{4}x - 2$
23. $P = 2l + 2w$ for l $l = \frac{P - 2w}{2}$ 24. $ax + by + c = 0$ for x $x = -\frac{by + c}{a}$ 25. $A = 0.5h(a + b)$ for b $b = \frac{2A}{h} - a$

Mixed Review Exercises

- Simplify.** 6. $3y + 16$ 7. $26ab$ 8. -4
1. $5(8 - 6 + 1)$ {15} 2. $(-4)^2 - 3^2$ {7} 3. $|5 + (-7)|$ {2} 4. $6(-\frac{1}{2})(-4)(-\frac{1}{3})$
5. $\frac{-12 \div 3}{-1 - 1}$ {2} 6. $9y - 2(3y - 8)$ 7. $3(4ab) - (2a)(-7b)$ 8. $(-c)^3(-d)^5$ c^3d^5
9. $3(x + 2y) + 4(-2x - y)$ $-5x + 2y$ 10. $-5p + \frac{2}{3}(9 - 3p) - 2$ $-7p + 4$ 11. $\frac{2m - 4}{-2}$ $-m + 2$

NAME _____ DATE _____

1–8 Words into Symbols

Objective: To translate word phrases into algebra expressions and word sentences into equations.

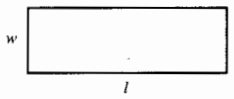
Vocabulary

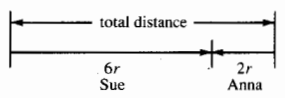
Uniform motion Motion at a constant speed. It is described by the formula
distance = rate \times time, or $d = rt$.

<p>Example 1 Represent each word phrase by an algebraic expression. Use n for the variable.</p> <p>a. Four less than twice a number b. The sum of twice a number and its square c. The difference between a number and three</p>	<p>Solution</p> <p>a. $2n - 4$ b. $2n + n^2$ c. $n - 3$</p>
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Represent each phrase by an algebraic expression. Use n for the variable.

1. Four less than the product of a number and three $3n - 4$
2. The quotient when eight is divided by twice a number $\frac{8}{2n}$
3. The square of the sum of a number and three $(n + 3)^2$
4. Five more than the reciprocal of a number $\frac{1}{n} + 5$

<p>Example 2 A rectangle has a width of w yards and a perimeter of 48 yards. Find the length in terms of w.</p>	<p>Solution Make a sketch. Call the length l. The perimeter of a rectangle is twice the length added to twice the width.</p> <div style="display: flex; align-items: center; justify-content: center;"> <div style="margin-right: 20px;"> $2l + 2w = 48$ $l + w = 24$ $l = 24 - w$ </div> <div style="text-align: center;">  </div> </div> <p>\therefore the length of the rectangle is $(24 - w)$ yd.</p>
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<p>Example 3 Anna and Sue rode toward each other on their bicycles. Sue's rate was three times Anna's rate of r mi/h. They met after two hours. How far apart were they before traveling? Express your answer in terms of r.</p>	<p>Solution The chart below shows all of the given information.</p> <table border="1" style="margin-bottom: 10px; border-collapse: collapse; width: 100%;"> <thead> <tr> <th></th> <th>Rate \times Time</th> <th>Distance</th> </tr> </thead> <tbody> <tr> <td>Anna</td> <td>$r \cdot 2$</td> <td>$2r$</td> </tr> <tr> <td>Sue</td> <td>$3r \cdot 2$</td> <td>$6r$</td> </tr> </tbody> </table> <div style="text-align: center;">  </div> <p>Their initial distance apart equals the sum of the distances traveled, or $8r$ mi.</p>		Rate \times Time	Distance	Anna	$r \cdot 2$	$2r$	Sue	$3r \cdot 2$	$6r$
	Rate \times Time	Distance								
Anna	$r \cdot 2$	$2r$								
Sue	$3r \cdot 2$	$6r$								

NAME _____ DATE _____

1–8 Words into Symbols (continued)

Express each answer in simplest form in terms of the given variable.

5. Carl is a years old. His sister Jenny is six more than twice his age. What is the average of their ages? $\frac{3}{2}a + 3$
6. The base and height of a triangle are consecutive odd integers, and the height exceeds the base. If the base is b cm, find the area of the triangle. $\frac{1}{2}b(b + 2)$ cm²
7. At 2:00 P.M. a train left a station traveling east at r mi/h. At 3:00 P.M. a second train headed west from the station at a rate 20 mi/h faster than the first. How far apart were the trains at 5:00 P.M.? $3r + 2(r + 20)$ mi \rightarrow $(5r + 40)$ mi
8. Seth sold 16 tickets to a school play. Of these, a were adults' tickets at \$3.75 each. The rest were children's tickets at \$2.50 each. How much money did Seth collect? $3.75a + 2.50(16 - a)$ dollars \rightarrow $(1.25a + 40)$ dollars

<p>Example 4 The number of dimes and the number of quarters that Erin has earned in tips are consecutive even integers. She has fewer dimes than quarters. The total value of her coins is \$7.50.</p>	<p>a. Choose a variable to represent the number of dimes. b. Write an expression for the number of quarters in terms of the number of dimes. c. Write an equation that describes the situation.</p>
<p>Solution</p>	<p>a. Let d = the number of dimes. b. Then $d + 2$ = the number of quarters. c. The value of the dimes is $0.10d$ dollars, and the value of the quarters is $0.25(d + 2)$ dollars. $\therefore 0.10d + 0.25(d + 2) = 7.50$, or $0.35d + 0.5 = 7.50$.</p>

Choose a variable to represent an unknown number, and then write an equation to describe the given situation.

9. In triangle ABC , the measure of $\angle A$ is four times that of $\angle B$. In addition, the measure of $\angle C$ is 15° less than half the measure of $\angle A$. $b + 4b + (2b - 15) = 180$
10. Carlos worked twice as many hours as Alan on Monday. After each boy worked nine more hours on Tuesday, the sum of their hours was three times the number of hours that Carlos worked on Monday. $(h + 9) + (2h + 9) = 3(2h)$
11. A class contributed \$27.50 in dimes and quarters to a local charity. There were 143 coins in all. $0.10d + 0.25(143 - d) = 27.50$
12. Two planes took off from a Chicago airport flying in opposite directions. One plane traveled 30 mi/h faster than the other. They were 1500 miles apart after 2 hours. $2r + 2(r + 30) = 1500$

1-9 Problem Solving with Equations

Objective: To solve word problems by using an equation in one variable.

Plan for Solving a Word Problem

- Step 1** Read the problem carefully a few times. Decide what numbers are asked for and what information is given. Making a sketch may be helpful.
- Step 2** Choose a variable and use it with the given facts to represent the number(s) described in the problem. Labeling your sketch or arranging the information in a chart may help.
- Step 3** Reread the problem. Then write an equation that represents relationships among the numbers in the problem.
- Step 4** Solve the equation and find the required numbers.
- Step 5** Check your results with the *original* statement of the problem. Give the answer.

CAUTION Problems can contain information that is unnecessary or contradictory. Sometimes there is not enough information given. Therefore, it is important to understand the given facts and their relationships before you try to solve a problem.

Example The Junior class sold shirts bearing the school insignia for \$12.00 each. An extra \$1.00 was charged to have a shirt monogrammed. There were 324 shirts sold, and a total of \$4036.00 was collected. Of the shirts sold, 174 were bought by Juniors. How many shirts were *not* monogrammed?

Solution

- Step 1** The problem asks for the number of shirts sold without a monogram.
- Step 2** Let x = the number of shirts sold without a monogram. Then $324 - x$ = number of shirts sold *with* a monogram.

	Price	× Number	= Sales
Without a monogram	12	x	$12x$
With a monogram	13	$324 - x$	$13(324 - x)$
Total Shirt Sales			4036

- Step 3** Sales without a monogram + Sales with a monogram = Total Sales
 $12x + 13(324 - x) = 4036$
- Step 4**

$$12x + 4212 - 13x = 4036$$

$$-x + 4212 = 4036$$

$$-x = -176$$

$$x = 176 \quad \text{(shirts without a monogram)}$$

$$324 - x = 148 \quad \text{(shirts with a monogram)}$$

(Solution continues on the next page.)

1-9 Problem Solving with Equations (continued)

Step 5 Check: Is the total number of shirts 324? $176 + 148 \stackrel{?}{=} 324$
 $324 = 324 \quad \checkmark$

Do shirt sales total \$4036.00? $176(12) + 148(13) \stackrel{?}{=} 4036$
 $2112 + 1924 \stackrel{?}{=} 4036$
 $4036 = 4036 \quad \checkmark$

\therefore 176 shirts were sold without a monogram.
 The information about the number of shirts bought by Juniors was unnecessary.

Solve each of the following problems. If there is not enough information to solve the problem, say so. If extra information is given, identify it.

- Cheryl's weekly allowance is \$2.00 more than Emily's. Together they get \$11.00. What is each girl's weekly allowance? **Cheryl: \$6.50; Emily: \$4.50**
- A child's bank contains 70 coins consisting of nickels and dimes that have a total value of \$5.55. How many of each kind of coin are there? **29 nickels, 41 dimes**
- A store sold 40 baseballs and 14 softballs over a two-week period. The sales for these items totaled \$200. What was the price of one baseball? **Not enough information.**
- The length of a rectangle is 6 cm more than the width. A square can be formed by tripling the width of the rectangle, and reducing its length by 2 cm. Find the dimensions of the rectangle. **8 cm × 2 cm**
- The perimeter of an isosceles triangle is 36 cm, and the area is 60 cm². The length of the base is 3 cm less than the length of a leg. Find the length of each side. **10 cm, 13 cm, 13 cm; the area is extra information.**
- The measures of the angles of a quadrilateral are consecutive odd integers. Find the measure of each angle. (*Hint:* The sum of the measures of the angles of a quadrilateral is 360°.) **87°, 89°, 91°, 93°**
- Two trains whose rates differ by 8 mi/h leave stations that are 432 miles apart at 10 A.M. If the trains meet at 2 P.M., what is the rate of each train? **50 mi/h, 58 mi/h**
- A hiker hikes up a mountain 1 mi/h slower than she hikes down the mountain. If it takes her 1.5 hours to hike up the mountain, and only 1 hour to hike down it, how fast does she move in each direction? **up: 2 mi/h; down: 3 mi/h**
- Sharon earned \$460 in simple interest on an investment of \$6200. Some of the money earned an interest rate of 5%, and the rest earned 8%. Her average rate of interest was 7.4%. How much did she invest at each rate? **\$1200 at 5%, \$5000 at 8%; the average rate of interest is extra information.**

Mixed Review Exercises

Solve.

- $4 - 5x = 14$ {-2}
- $4(y - 3) = y - 15$ {-1}
- $7z + 4 = 4z + 16$ {4}

Evaluate each expression if $a = -5$ and $b = 10$.

- $(a + b)^2$ 25
- $(ab)^2$ 2500
- $|a - 2b|$ 25
- $\frac{b - a}{-3}$ -5
- $\frac{ab}{-2}$ 25
- $\frac{b \div a}{2}$ -1