A TECHNIQUE FOR INTEGRATION BY PARTS

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When we first encounter integration by parts we are confronted with the usual formula:

$$\int u\,dv = uv - \int v\,du.$$

The expressions u and dv are chosen with two criteria in mind:

(1) v should be easy to find from dv;
(2) the integral $\int v\,du$ should, in some sense, be "better" or easier than $\int u\,dv$.

The difficulty usually arises in choosing u and dv to satisfy these conditions. An acronym gives us a method of selection which helps to satisfy criterion (2) above. The word is LIATE, standing for *L*ogarithmic *I*nverse trig *A*lgebraic *T*rig *E*xponential. These five basic types of functions are those encountered in our integral.

If, for example, our integral involves the product of a logarithmic function and an algebraic function, we choose u to be the log part and, keeping criterion (1) in mind, the algebraic part is chosen for dv. By making this selection, in $\int v\,du$, the logarithmic character has disappeared by virtue of the differentiation. The antidifferentiation of the algebraic part results (most of the time) in an algebraic v. Consequently, the integral $\int v\,du$ is strictly algebraic in its integrand and hopefully easier than our original integral.

Similarly, if we have any combination of two of these types of functions in our original integral, we choose for u that type that appears first in LIATE and dv is whatever is left, that part that appears second in LIATE.

Rationale: Differentiating logs and inverse trig functions changes their character to algebraic, while algebraic functions run the "middle of the road;" differentiating them yields algebraic, while integration sometimes yields nonalgebraic, so we would probably differentiate them before integrating. Trig and exponential functions "bring up the rear" since their differentiation or antidifferentiation results in similar functions.

As an example, consider $\int x \ln(x)\,dx$. Using LIATE, we pick $u = \ln(x)$ and $dv = x\,dx$. We obtain $du = x^{-1}\,dx$ and $v = x^2/2$. Consequently,

$$\int x \ln(x)\,dx = \frac{x^2}{2}\ln(x) - \int \frac{x}{2}\,dx$$

$$= \frac{x^2}{2}\ln(x) - \frac{x^2}{4} + C.$$

Students have liked using LIATE and find that choosing u and dv is much less frightening. The author has yet to see an integral done by integration by parts to which LIATE cannot be successfully applied.

To be able to use integration by parts, perhaps it is better LIATE than never.

Reference

1. Howard Anton, Calculus with Analytic Geometry, Wiley, New York, 1980.