NAME ______________________________ DATE ______________

12–3 The Quadratic Formula

Objective: To learn the quadratic formula and use it to solve equations.

The Quadratic Formula

The solutions of a quadratic equation in the form of $ax^2 + bx + c = 0$, $a \neq 0$ and $b^2 - 4ac \geq 0$ are given by the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Example 1 Use the quadratic formula to solve $3x^2 + 5x - 2 = 0$.

Solution

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where $a = 3$, $b = 5$, and $c = -2$.

$x = \frac{-5 \pm \sqrt{(5)^2 - 4(3)(-2)}}{2(3)}$ Substitute the given values of a, b, and c.

$= \frac{-5 \pm \sqrt{25 + 24}}{6}$

$= \frac{-5 \pm \sqrt{49}}{6} = \frac{-5 \pm 7}{6}$

$x = \frac{-5 + 7}{6} = \frac{2}{6} = \frac{1}{3}$ or $x = \frac{-5 - 7}{6} = \frac{-12}{6} = -2$

The check is left to you. The solution set is $\left\{\frac{1}{3}, -2\right\}$.

Use the quadratic formula to solve each equation.

1. $x^2 + 3x - 10 = 0$
2. $x^2 - 8x + 7 = 0$
3. $x^2 + 2x - 3 = 0$
4. $x^2 - 14x + 24 = 0$
5. $n^2 + 5n - 6 = 0$
6. $x^2 - 6x - 40 = 0$
7. $2x^2 + 3x - 2 = 0$
8. $3u^2 - 5u - 2 = 0$
9. $3x^2 - 10x - 8 = 0$
10. $3x^2 - 2x - 1 = 0$
11. $2x^2 - 5x - 7 = 0$
12. $5x^2 + 6x - 8 = 0$

Example 2 Use the quadratic formula to solve $x^2 = x - 6$.

Solution

$x^2 - x + 6 = 0$ Rewrite the equation in standard form.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where $a = 1$, $b = -1$, and $c = 6$.

$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(6)}}{2(1)} = \frac{1 \pm \sqrt{1 - 24}}{2} = \frac{1 \pm \sqrt{-23}}{2}$

Since $\sqrt{b^2 - 4ac} = \sqrt{-23}$ and $\sqrt{-23}$ isn't a real number, there is *no real solution.*

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12–3 The Quadratic Formula *(continued)*

Use the quadratic formula to solve each equation.

13. $x^2 - 4x + 6 = 0$ **14.** $2x^2 = 3x - 1$ **15.** $x^2 - 4x = 30$

16. $2x^2 + 2x + 5 = 0$ **17.** $4x^2 + 20x = -9$ **18.** $3x^2 - 3x + 4 = 0$

Example 3 Use the quadratic formula to solve $2x^2 - 3x - 4 = 0$. Give irrational roots in simplest radical form and then approximate them to the nearest tenth. You may wish to use a calculator.

Solution $2x^2 - 3x - 4 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \text{ where } a = 2, b = -3, \text{ and } c = -4.$$

$$x = \frac{3 \pm \sqrt{9 - 4(2)(-4)}}{2(2)} \quad \text{Substitute the given values of } a, b, \text{ and } c.$$

$$= \frac{3 \pm \sqrt{9 + 32}}{4} \quad \text{Simplify.}$$

$$= \frac{3 \pm \sqrt{41}}{4}$$

Since $\sqrt{41} \approx 6.40$, $x \approx \frac{3 + 6.4}{4} = 2.35 \approx 2.4$

or $x \approx \frac{3 - 6.4}{4} = -0.85 \approx -0.9$

The check is left to you.

The solution set is $\left\{\frac{3 + \sqrt{41}}{4}, \frac{3 - \sqrt{41}}{4}\right\}$ or $\{2.4, -0.9\}$.

Use the quadratic formula to solve each equation. Give irrational roots in simplest radical form and then approximate them to the nearest tenth. You may wish to use a calculator.

19. $2x^2 = 8x - 5$ **20.** $3x^2 + 2x = 2$ **21.** $x^2 - 4x - 10 = 0$

22. $x^2 - 4x - 2 = 0$ **23.** $2x^2 - 4x + 1 = 0$ **24.** $3x^2 - 8x + 2 = 0$

25. $2x^2 + 1 = 3x$ **26.** $3x^2 + x = 2$ **27.** $4x^2 - 11x = 3$

Mixed Review Exercises

Solve each open sentence and graph its solution set.

1. $|x - 2| \le 5$ **2.** $2|y + 5| = 4$ **3.** $|2n + 3| < 5$

4. $1 < 2z + 1 \le 7$ **5.** $\sqrt{x} = 5$ **6.** $\sqrt{5n + 1} = 6$

7. $2\sqrt{2x} = 12$ **8.** $|3 + 2k| = 11$ **9.** $3|2 - m| = 12$

Solve by completing the square.

10. $x^2 - 8x + 12 = 0$ **11.** $3x^2 + 6x = 0$ **12.** $c^2 - c = 12$