Ben Bryce Pg 86 10/1/18 Quiz 8

Problem 6: If a rock is thrown upward on the Planet Mars with a velocity of 10 m/s, its height in Meters t seconds late is given by $y = 10t - 1.86t^2$

a.) Find the average velocity for the time Period beginning when $t = 2$ and lasting

(i) 0.5 seconds
(ii) 0.1 seconds
(iii) 0.05 seconds
(iv) 0.01 seconds
(v) 0.001 seconds

(i) Let $\bar{V}$ mean the average velocity. $\bar{V} = \frac{\Delta y}{\Delta t}$, where Δy denotes the change in height and Δt is the elapsed time. At time $t = 1$ second;

$$y = 10(1) - 1.86(1)^2 \rightarrow y = 10 - 1.86$$

$$y = 8.14 \text{ m/s}$$

at $t = 2$ seconds, $y = 10(2) - 1.86(2)^2$

$$y = 12.56 \text{ m/s}$$

$$\bar{V} = \frac{12.56 - 8.14}{2 - 1} \qquad \boxed{\bar{V} = 4.42 \text{ m/s}}$$

(ii) 1.5 seconds

$$\bar{V} = \frac{10(1.5) - 1.86(1.5)^2 - 8.14}{1.5 - 1} \rightarrow \boxed{\bar{V} = 5.35 \text{ m/s}}$$

(iii) 1.1 seconds

$$\bar{V} = \frac{10(1.1) - 1.86(1.1)^2 - 8.14}{1.1 - 1} \rightarrow \boxed{\bar{V} = 6.094 \text{ m/s}}$$

(iv) 1.01

$$\bar{V} = \frac{10(1.01) - 1.86(1.01)^2 - 8.14}{1.01 - 1} \rightarrow \boxed{\bar{V} = 6.2614 \text{ m/s}}$$

(v) 1.001

$$\bar{V} = \frac{10(1.001) - 1.86(1.001)^2 - 8.14}{1.001 - 1} \rightarrow \boxed{\bar{V} = 6.27814 \text{ m/s}}$$

b.) Estimate the velocity when $t = 1$

Found when Plugged in above, the closer Δt gets to 0, the closer $\bar{V}$ gets to 6.28 m/s.

4.42, 5.35, 6.094, 6.2614, 6.278 ....

It appears that $\bar{V} \rightarrow 6.28$ m/s as $\Delta t \rightarrow 0$.

Thus, I estimate that the instantaneous velocity is 6.28 m/s.

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Reworked Problem:

$$\bar{V} = \frac{10(1+\Delta t) - [1.86(1+\Delta t)^2] - 8.14}{(1+\Delta t) - 1} = \bar{V} = \frac{10 + 10\Delta t - [1.86(1 + 2\Delta t + \Delta t^2)] - 8.14}{\Delta t}$$

$$\bar{V} = \frac{10 + 10\Delta t - 1.86 - 3.72\Delta t - 1.86\Delta t^2 - 8.14}{\Delta t}$$

$$\bar{V} = \frac{6.28\Delta t - 1.86\Delta t^2}{\Delta t}$$

$$V = \frac{\Delta t(6.28 - 1.86\Delta t)}{\Delta t}$$

$$V = 6.28 - 1.86\Delta t$$

Thus, the average velocity between times 1 and $(1+\Delta t)$ is $\bar{V} = 6.28 - 1.86\Delta t$

So:

(i) from 1 to 2 seconds,

$\bar{V} = 6.28 - 1.86(1)$

$\bar{V} = 4.42$ m/s

(ii) from 1 to 1.5 seconds

$\bar{V} = 6.28 - 1.86(.5)$

$\bar{V} = 5.35$ m/s

(iii) from 1 to 1.1 seconds

$\bar{V} = 6.28 - 1.86(.1)$

$\bar{V} = 6.094$ m/s

(iv) from 1 to 1.01 seconds

$\bar{V} = 6.28 - 1.86(.01)$

$\bar{V} = 6.2614$ m/s

(v) from 1 to 1.001 seconds

$\bar{V} = 6.28 - 1.86(.001)$

$\bar{V} = 6.27814$ m/s

Symbolically, this is expressed as $\lim_{\Delta t \to 0} (6.28 - 1.86\Delta t) = 6.28$

This is defined as the instantaneous velocity of the rock at $t = 1$ second.