

## 8-7 Functions Defined by Equations

**Objective:** To define a function by using equations.

### Vocabulary

**Arrow notation** A notation involving an arrow used to define a function.

For example,  $P: n \rightarrow 5n - 500$ .

**Functional notation** A notation involving an equals sign used to define a function.

For example,  $P(n) = 5n - 500$ .

**Values of a function** Members of the range of the function.

**Symbols**  $g(2) = 6$  (Read “ $g$  of 2 equals 6” or “the value of  $g$  at 2 is 6.”)

**CAUTION**  $g(2)$  is not the product of  $g$  and 2. It names the number that  $g$  assigns to 2.

**Example 1** List the range of  $g: x \rightarrow x^2 - x - 6$  if the domain  $D = \{-2, -1, 0, 1, 2\}$ .

**Solution** In  $x^2 - x - 6$  replace  $x$  with each member of  $D$  to find the members of the range  $R$ .

$x$	$x^2 - x - 6$
-2	$(-2)^2 - (-2) - 6 = 0$
-1	$(-1)^2 - (-1) - 6 = -4$
0	$(0)^2 - (0) - 6 = -6$
1	$(1)^2 - (1) - 6 = -6$
2	$(2)^2 - (2) - 6 = -4$

$$R = \{0, -4, -6\}$$

*Note:* The function  $g$  assigns  $-4$  to both  $-1$  and  $2$ , and  $-6$  to both  $0$  and  $1$ . In listing the range of  $g$ , you name  $-4$  and  $-6$  only once each.

**Find the range of each function.**

1.  $g: x \rightarrow 2x + 1, D = \{-1, 0, 1\}$

2.  $f: x \rightarrow 3x - 2, D = \{1, 2, 3\}$

3.  $h: x \rightarrow 1 - 4x, D = \{-2, 0, 2\}$

4.  $h(y) = 3y + 1, D = \{-3, 0, 1\}$

5.  $G: a \rightarrow 3a - 2, D = \{-2, 0, 2\}$

6.  $F(x) = 2 - 4x, D = \{-1, 0, 1\}$

7.  $F(x) = 5x - 4, D = \{-1, 2, 3\}$

8.  $Q(n) = 4n - 3, D = \{0, 2, 3\}$

9.  $P(z) = z^2 - 2z, D = \{-1, 0, 1\}$

10.  $H: b \rightarrow b^2 - b - 2, D = \{-1, 0, 2\}$

11.  $g: x \rightarrow x^2 + 3x - 4, D = \{-1, 2, 4\}$

12.  $f: x \rightarrow x^2 - x - 6, D = \{-2, 0, 3\}$

13.  $F(x) = x^3 + x^2 + 2x, D = \{-1, 0, 1\}$

14.  $N(a) = a^3 - 2a^2 + 3a, D = \{0, 2, 3\}$

**8-7 Functions Defined by Equations** (continued)**Example 2** Given  $f: x \rightarrow x^2 - x$  with the set of real numbers as the domain. Find:

a.  $f(2)$     b.  $f(-3)$     c.  $f(4)$

**Solution** First write the equation:  $f(x) = x^2 - x$ 

Then substitute:    a.  $f(2) = 2^2 - 2 = 4 - 2 = 2$

b.  $f(-3) = (-3)^2 - (-3) = 9 + 3 = 12$

c.  $f(4) = 4^2 - 4 = 16 - 4 = 12$

Find the values for each given function with the set of real numbers as the domain.

15.  $f(x) = 3x - 2$     a.  $f(2)$     b.  $f(-2)$     c.  $f(-4)$

16.  $p(x) = 4 - 2x$     a.  $p(1)$     b.  $p(0)$     c.  $p(-2)$

17.  $R: t \rightarrow t + 2$     a.  $R(2)$     b.  $R(-1)$     c.  $R(-3)$

18.  $G: n \rightarrow n - 3$     a.  $G(0)$     b.  $G(2)$     c.  $G(-3)$

19.  $h(a) = 2a^2 + 1$     a.  $h(3)$     b.  $h(-2)$     c.  $h(0)$

20.  $k(t) = 2t^2 - 3$     a.  $k(4)$     b.  $k(-2)$     c.  $k(-3)$

21.  $g(x) = x^2 - 1$     a.  $g(4)$     b.  $g(-4)$     c.  $g(0)$

22.  $h(y) = 3y^2 + 1$     a.  $h(2)$     b.  $h(-2)$     c.  $h(-1)$

23.  $R: y \rightarrow y^3 + 2$     a.  $R(0)$     b.  $R(-2)$     c.  $R(2)$

24.  $N: t \rightarrow t^3 - 8$     a.  $N(3)$     b.  $N(-3)$     c.  $N(0)$

25.  $f: x \rightarrow x^2 + 2x$     a.  $f(-2)$     b.  $f(2)$     c.  $f(-1)$

26.  $g: t \rightarrow 3t^2 - 2t$     a.  $g(3)$     b.  $g(1)$     c.  $g(-1)$

27.  $P(y) = y - y^2$     a.  $P(2)$     b.  $P(0)$     c.  $P(-2)$

**Mixed Review Exercises**

Simplify.

1.  $\frac{3n-1}{2n^2} + \frac{2}{n}$

2.  $3\frac{1}{3} + 2\frac{3}{4} + 5\frac{2}{3} + 1\frac{1}{4}$

3.  $(-12)\left(\frac{x}{4}\right)$

4.  $(-2)(3a + 2b - c)$

5.  $-[8 + (-3)]$

6.  $2(3m - 5)$

7.  $-80\left(\frac{1}{4}\right)\left(\frac{1}{5}\right)$

8.  $\frac{2e^2f}{3ef^2} \cdot \frac{6de^2}{8ef}$

9.  $\frac{x^2 - 4}{x^2 + 4x + 4}$