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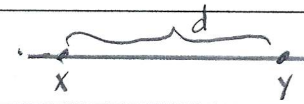
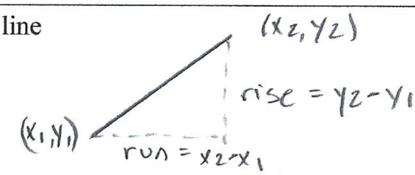
ACT/SAT Formulas and Facts

revised 4/2024

Algebra

absolute value $\lvert x-y\rvert$	the distance between 2 points, x and y, on a number line 
absolute value $\lvert x\rvert$	$\lvert x\rvert = \begin{cases} x, & if\ x \geq 0 \\ -x, & if\ x < 0 \end{cases}$ $\lvert x\rvert$ expresses the distance from $x$to zero on the number line
distance formula	$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$= distance between 2 points (x_1, y_1) and (x_2, y_2) on coordinate plane *Note that the distance formula is derived from the Pythagorean Theorem*
midpoint formula	$M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ = $b^2 - 4ac$(average of the x coordinates, average the y coordinates)
slope of a line between any 2 points (x_1, y_1) and (x_2, y_2)	$m = \frac{y_2-y_1}{x_2-x_1}$ slope is the measure of the steepness of a line  **horizontal line** slope = 0 since rise = 0 **vertical line** slope is undefined since run = 0 **parallel lines** have equal slopes; **perpendicular lines -** slopes are opposite (negative) reciprocals; product of slopes = -1
linear equation slope intercept form point-slope form standard form	 y = mx + b, where m = slope and (0, b) = y-intercept $y - y_1 = m(x - x_1)$ Ax + By = C
factoring difference of 2 squares sum of 2 squares difference of 2 cubes sum of 2 cubes perfect square trinomials	 $a^2 - b^2 = (a+b)(a-b)$ prime (not factorable) $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$ $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$ $(a+b)^2 = a^2 + 2ab + b^2$ and $(a-b)^2 = a^2 - 2ab + b^2$ 1 1 1 1 2 1 1 3 3 1 1 4 6 4 1 1 5 10 10 5 1
quadratic formula	$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ yields solutions for $ax^2 + bx + c = 0$ Note: the sum of the two solutions is $\frac{-b}{a}$ and product of solutions = $\frac{c}{a}$
discriminant test	$b^2 - 4ac = 0$ ➔ one real solution $b^2 - 4ac > 0$ ➔ two real solutions; if $b^2 - 4ac$ is a perfect square, the solutions are rational $b^2 - 4ac < 0$ ➔ two complex conjugate solutions
exponent rules	$x^0 = 1$ $(x^a)^b = x^{ab}$ $x^a x^b = x^{a+b}$ $\frac{x^a}{x^b} = x^{a-b}$ $x^{-a} = \frac{1}{x^a}$ and $\frac{1}{x^{-a}} = x^a$
radicals and exponents	$\sqrt{x} = x^{\frac{1}{2}}$ $\sqrt[3]{x} = x^{\frac{1}{3}}$ $\sqrt[b]{x^a} = x^{\frac{a}{b}}$
complex number	$i = \sqrt{-1}$ $i^2 = -1$ $i^3 = -i$ $i^4 = 1$
absolute value equation	If |**algebraic expression**| = A, then set up two equations and solve: **algebraic expression** = A and **algebraic expression** = -A
absolute value inequalities	If |algebraic expression| < A, then set up compound inequality (conjunction) and solve: **-A < algebraic expression < A** If |algebraic expression| > A, then set up two inequalities (disjunction) and solve: **algebraic expression > A *or* algebraic expression < -A**

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Distance, rate, time	$d = rt$ distance = rate x time
arithmetic mean= average	$average = \frac{x_1+x_2+x_3+\cdots+x_n}{n}$
median	The median of a set of numbers is the middle value when the data are in order. If there is an even number of terms, the median is the average of the two in the middle.
mode	The mode of a set of numbers is the number that occurs most frequently
% change	$percent\ change = \frac{change\ in\ quantity}{original\ quantity}$
proportion	$\frac{a}{b} = \frac{c}{d}$ $implies\ ad = bc$ by cross-multiplication
direct variation	$y = kx$ or $\frac{x_1}{y_1} = \frac{x_2}{y_2}$ where $k$is the constant of variation
inverse variation	$xy = k$ or $x_1 y_1 = x_2 y_2$ where $k$is the constant of variation
arithmetic sequence	$a_n = a_1 + (n-1)d$, nth term of arithmetic sequence, d is the common difference between terms
arithmetic series	$S_n = \frac{n(a_1+a_n)}{2}$, S_n = sum of first "n" terms of a finite geometric series; a_1 = first term
geometric sequence	$a_n = a_1 r^{n-1}$ nth term of the geometric series; r is the common ratio
geometric series	$S_n = a_1 + a_1 r + \cdots + a_1 r^{n-1} = \frac{a_1(1-r^n)}{1-r}$, S_n = sum of first "n" terms of geometric series
probability	Probability of an outcome happening = $\frac{\text{number of desired outcomes}}{\text{total number of possible outcomes}}$ Probability of two mutually exclusive events, A and B, happening =$P(A) \bullet P(B)$
combinations	A combination means the order of the elements doesn't matter. For example, a shirt and pants is the same thing as pants and a shirt. Possible combinations of 3 shirts and 4 pants = 3 x 4 = 12. $nC_r = \frac{n!}{r!(n-r)!}$ number of combinations of n items taken $r$at a time; order does not matter
permutations	$nP_r = \frac{n!}{(n-r)!}$ number of ways to arrange n items taken $r$at a time; order does matter
logarithms	$log_b\, y = x$ $means$ $y = b^x$ $log_b\, M\, N = log_b\, M + log_b\, N$ $log_b \left(\frac{M}{N}\right) = log_b\, M - log_b\, N$ $log_b\, M^N = N\, log_b\, M$ $log_b\, M = \frac{log_c\, M}{log_c\, b}$ (change of base formula)
Transformations Translation (no rotation or size change)	(x, y) → (x + a, y + b) represents horizontal shift of "a" units, vertical shift of "b" units);
Reflection (flip)	
Over x-axis	(x, y) → (x, -y)
Over y-axis	(x, y) → (-x, y)
Over line y = x	(x, y) → (y, x)
Over origin (line y= -x)	(x, y) → (-y, -x)
Rotation about origin	
90° CCW or 270° CW	(x, y) → (-y, x)
180° CCW or 180° CW	(x, y) → (-x, -y)
270° CCW or 90° CW	(x, y) → (y, -x)

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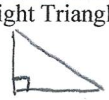
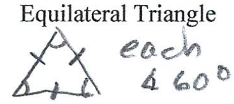
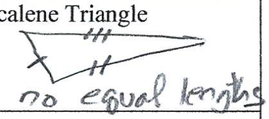
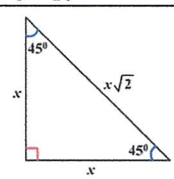
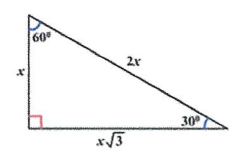
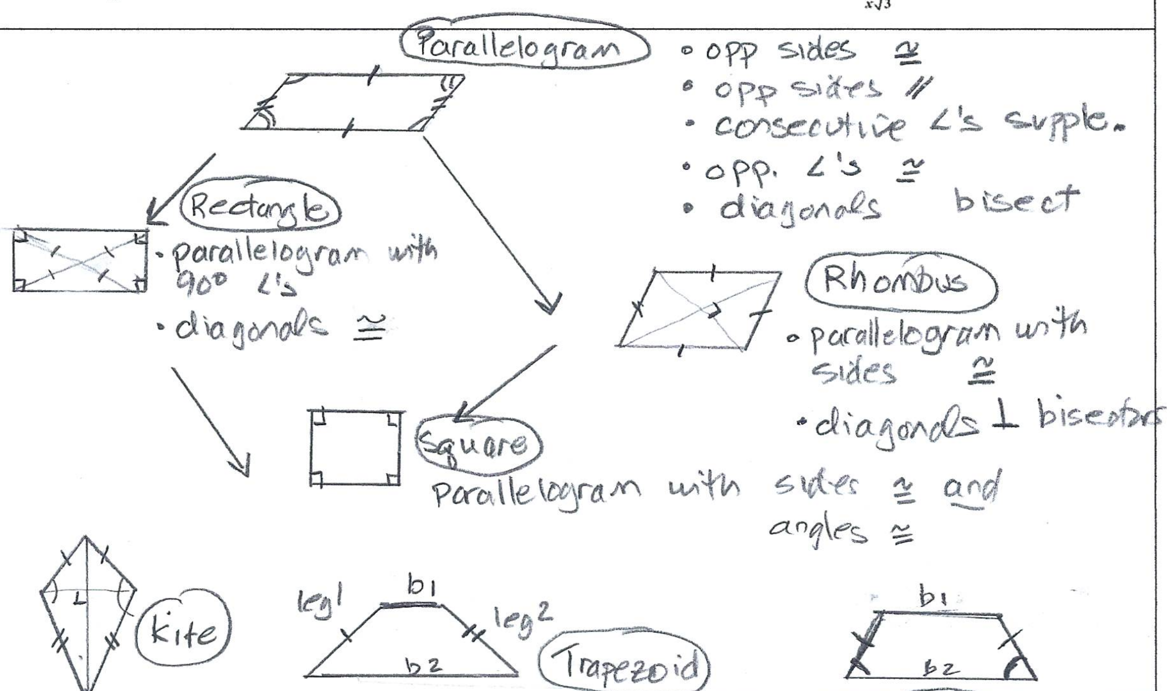
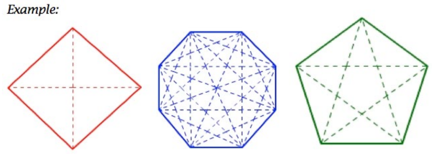
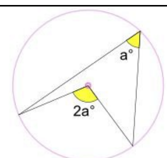
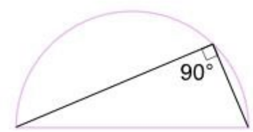
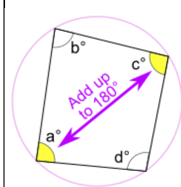
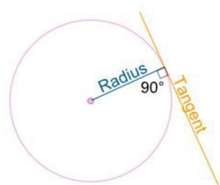
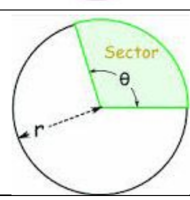
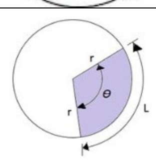
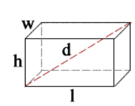
Geometry

Perimeter	**In general, perimeter = sum of lengths of sides**
square	P = 4s
rectangle	P = 2L + 2W
circle	C = 2πr (circumference)
Area	**Note that the units are square units.**
square	$A = s^2$
rectangle, parallelogram	A = bh (note base and height are ***always*** perpendicular)
triangle	A = ½ bh
kite	A = ½ (d_1 d_2) where d_1 and d_2 are lengths of the diagonals
circle	$A = \pi r^2$
trapezoid	A = ½ (b_1 + b_2)h (average of bases times the height)
Volume	**B = area of base; h = height**
cube	$V = Bh = s^3$ where s is the side length
rectangular prism	$V = Bh = lwh$
cylinder	$V = Bh = \pi r^2 h$
Triangles	
	Right Triangle  Isosceles triangle  Equilateral Triangle  Scalene Triangle 
Congruency Theorems	SSS, SAS, ASA, or AAS or use H-L (right triangles only)
Pythagorean Theorem	$c^2 = a^2 + b^2$ is used to find length of sides or hypotenuse, c, for a right triangle If $c^2 = a^2 + b^2$, then the triangle is a right triangle (Converse of Pythagorean Theorem) If $c^2 < a^2 + b^2$, then the triangle is acute If $c^2 > a^2 + b^2$, then the triangle is obtuse
Common Pythagorean Triples	3 – 4 – 5 5 – 12 – 13 7 – 24 – 25 8 – 15 – 17 6 – 8 – 10 10 – 24 – 26 14 – 48 – 50 16 – 30 – 34
Special Triangles	 45-45-90 triangle (Isosceles right triangle) 30-60-90 triangle 
Quadrilaterals	

one set of opp sides //

Isosceles Trapeziod

Other Formulas and Facts	
sum of interior angles of a convex polygon	S = (n – 2)180° where n = # of sides (triangle sum = 180°; quadrilateral sum = 360°; pentagon sum = 540°; hexagon sum = 720°)
sum of exterior angles in a convex polygon	sum of exterior angles always equals 360°
number of diagonals in a convex polygon	Example:  # of diagonals $= \frac{n(n-3)}{2}$, where n = number of sides
inscribed angle facts	 An inscribed angle a is half the central angle, 2a. Therefore, the inscribed angle 90° is half of the central angle 180°.   A Cyclic Quadrilateral's opposite angles add up to 180°: a + c = 180° b + d = 180°  A tangent is a line that just touches a circle at one point. It always forms a right angle with the circle's radius to the point of tangency.
sector area of a circle	 $A = \frac{\theta^o}{360^o}\pi r^2$= fractional part of the circle's area
length of intercepted arc	 $L = \frac{\theta^o}{360^o}2\pi r$ = fractional part of the circumference
Length of diagonal of a rectangular prism	$d = \sqrt{L^2 + H^2 + W^2}$ 

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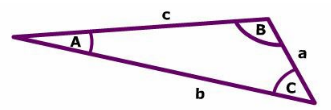
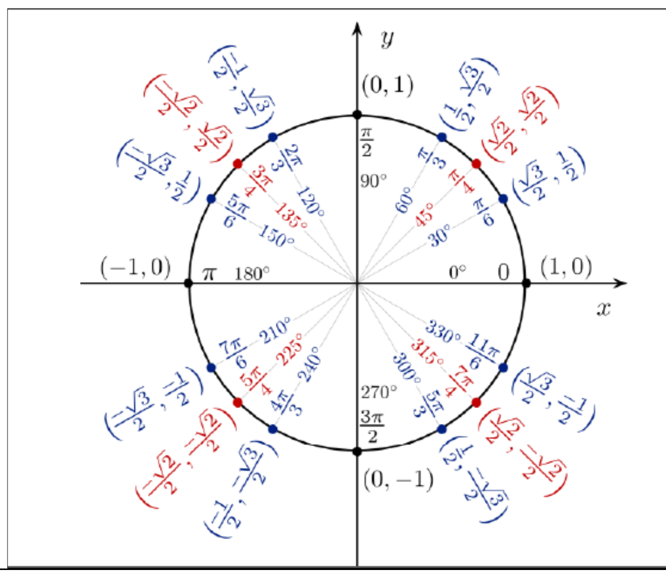
Trigonometry

Trigonometric ratios	$\sin\ \Theta = \frac{opp}{hyp} = \frac{y}{r}$ $\qquad$ $\csc\ \Theta = \frac{1}{\sin\Theta} = \frac{hyp}{opp} = \frac{r}{y}$ $\cos\ \Theta = \frac{adj}{hyp} = \frac{x}{r}$ $\qquad$ $\sec\ \Theta = \frac{1}{\cos\Theta} = \frac{hyp}{adj} = \frac{r}{x}$ $\tan\ \Theta = \frac{\sin\Theta}{\cos\Theta} = \frac{opp}{adj} = \frac{y}{x}$ $\qquad$ $\cot\ \Theta = \frac{\cos\Theta}{\sin\Theta} = \frac{adj}{opp} = \frac{x}{y}$
Identities	$\sin^2\Theta + \cos^2\Theta = 1$ $\qquad$ $1 + \cot^2\Theta = \csc^2\Theta$ $\qquad$ $1 + \tan^2\Theta = \sec^2\Theta$
Law of Sines and Law of Cosines (for non-right triangles)	For **non**-right triangles, use the Law of Sines to find side lengths and angles when possible, or use Law of Cosines when you have 2 sides and the included angle .  $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ $\qquad$ $c^2 = a^2 + b^2 - 2ab\ \cos C$
Sine & Cosine functions	$y = A\sin(Bx + C) + D$ $y = A\cos(Bx + C) + D$ $\qquad$ $Amplitude = \lvert A\rvert$ $\qquad$ $Period = \frac{2\pi}{B}$ $Horizontal\ shift = -\frac{C}{B}$ $\qquad$ $Vertical\ shift = D$
Unit Circle $x = \cos\theta$ $y = \sin\theta$ $y/x = \tan\theta$ 180° = π radians	 Degrees $\quad$ radians 0 $\quad$ 0 30 $\quad$ $\frac{\pi}{6}$ 45 $\quad$ $\frac{\pi}{4}$ 60 $\quad$ $\frac{\pi}{3}$ 90 $\quad$ $\frac{\pi}{2}$ 180 $\quad$ π 270 $\quad$ $\frac{3\pi}{2}$ 360 $\quad$ 2π

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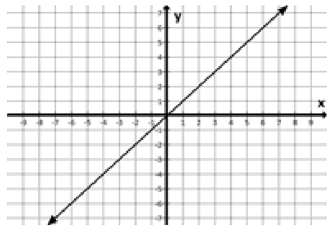
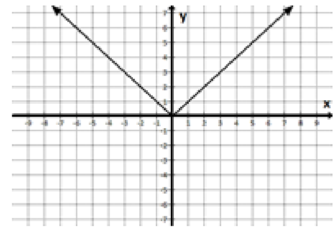
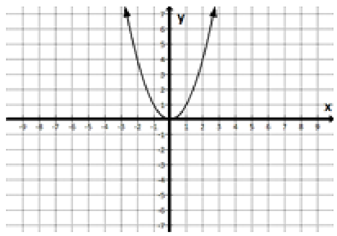
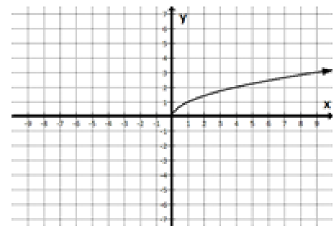
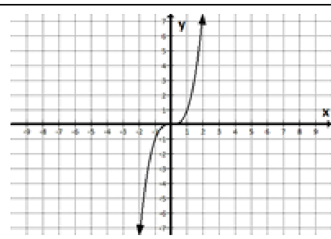
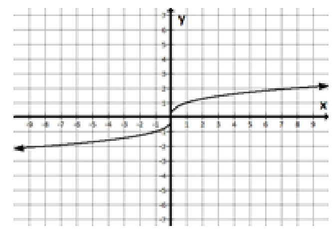
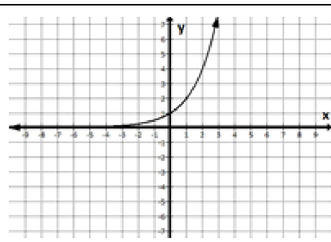
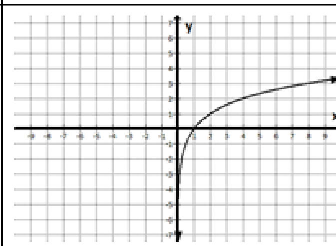
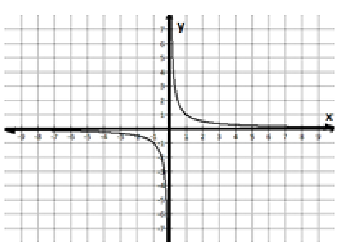
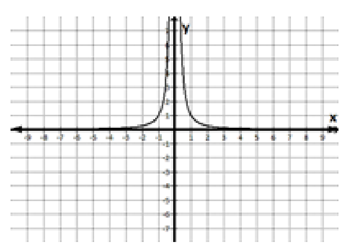
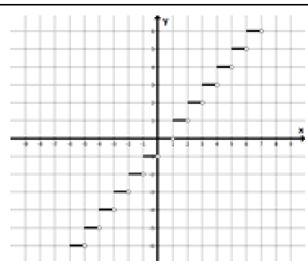
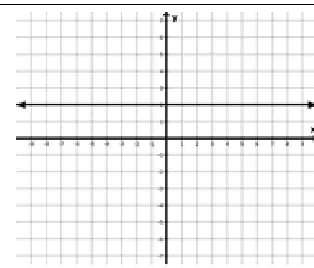
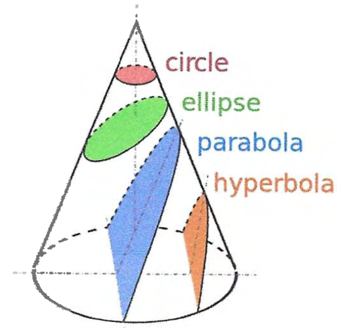
Graphs

Parent Function	Graph	Parent Function	Graph
$y = x$ **Linear**, Odd Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$ End Behavior: $x \to -\infty,\ y \to -\infty$ $x \to \infty,\ y \to \infty$		$y = \lvert x \rvert$ **Absolute Value**, Even Domain: $(-\infty, \infty)$ Range: $[0, \infty)$ End Behavior: $x \to -\infty,\ y \to \infty$ $x \to \infty,\ y \to \infty$	
$y = x^2$ **Quadratic**, Even Domain: $(-\infty, \infty)$ Range: $[0, \infty)$ End Behavior: $x \to -\infty,\ y \to \infty$ $x \to \infty,\ y \to \infty$		$y = \sqrt{x}$ **Radical**, Neither Domain: $[0, \infty)$ Range: $[0, \infty)$ End Behavior: $x \to \infty,\ y \to \infty$	
$y = x^3$ **Cubic**, Odd Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$ End Behavior: $x \to -\infty,\ y \to -\infty$ $x \to \infty,\ y \to \infty$		$y = \sqrt[3]{x}$ **Cube Root**, Odd Domain: $(-\infty, \infty)$ Range: $(-\infty, \infty)$ End Behavior: $x \to -\infty,\ y \to -\infty$ $x \to \infty,\ y \to \infty$	
$y = b^x,\ b > 1$ **Exponential**, Neither Domain: $(-\infty, \infty)$ Range: $(0, \infty)$ End Behavior: $x \to -\infty,\ y \to 0$ $x \to \infty,\ y \to \infty$		$y = \log_b(x),\ b > 1$ **Log**, Neither Domain: $(0, \infty)$ Range: $(-\infty, \infty)$ End Behavior: $x \to 0^+,\ y \to -\infty$ $x \to \infty,\ y \to \infty$	
$y = \frac{1}{x}$ **Rational (Inverse)**, Odd Domain: $(-\infty, 0) \cup (0, \infty)$ Range: $(-\infty, 0) \cup (0, \infty)$ End Behavior: $x \to -\infty,\ y \to 0$ $x \to \infty,\ y \to 0$		$y = \frac{1}{x^2}$ **Rational (Inverse Squared)**, Even Domain: $(-\infty, 0) \cup (0, \infty)$ Range: $(0, \infty)$ End Behavior: $x \to -\infty,\ y \to 0$ $x \to \infty,\ y \to 0$	
$y = \text{int}(x) = [x]$ **Greatest Integer**, Neither Domain: $(-\infty, \infty)$ Range: $\{y : y \in \mathbb{Z}\}$ (integers) End Behavior: $x \to -\infty,\ y \to -\infty$ $x \to \infty,\ y \to \infty$		$y = C$ (y = 2 in the graph) **Constant**, Even Domain: $(-\infty, \infty)$ Range: $\{y : y = C\}$ End Behavior: $x \to -\infty,\ y \to C$ $x \to \infty,\ y \to C$	

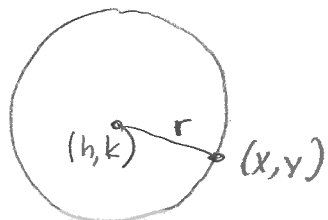
Conic Sections



CIRCLE

centered at (0,0) $x^2 + y^2 = r^2$

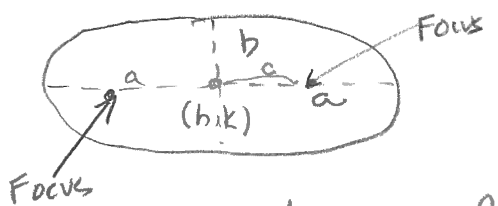
centered at (h,k) $(x-h)^2 + (y-k)^2 = r^2$



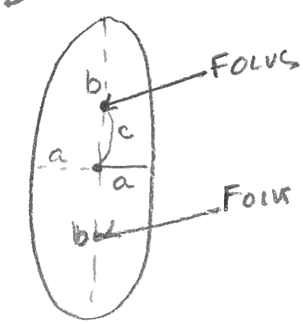
ELLIPSE

centered at (0,0) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

centered at (h,k) $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$



major axis = long axis



(Focus is always on the major axis)

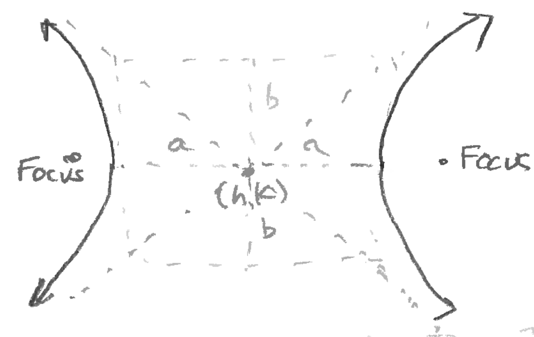
c = distance from center to focus $c^2 = a^2 - b^2$ or $b^2 - a^2$

HYPERBOLA

centered at (0,0) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

centered at (h,k) $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

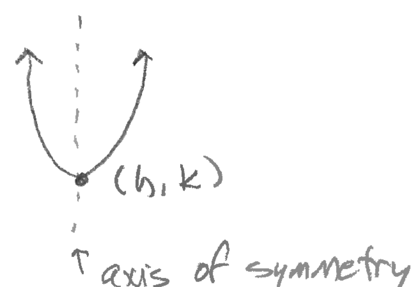
c = distance from center to Focus $c^2 = a^2 + b^2$



PARABOLA

$y = A(x-h)^2 + k$ vertex at (h,k)

or $y = ax^2 + bx + c$ vertex at $\left(-\frac{b}{2a}, f\left(\frac{-b}{2a}\right)\right)$



$a > 0$ opens up

$a < 0$ opens down