

5-7 Factoring Pattern for $x^2 + bx + c$, c positive

Objective: To factor quadratic trinomials whose quadratic coefficient is 1 and whose constant term is positive.

Vocabulary/Patterns

Factoring patterns for $x^2 + bx + c$ when c is positive:

When b is positive: $(x + ?)(x + ?)$

When b is negative: $(x - ?)(x - ?)$

Prime polynomial A polynomial with integral coefficients whose greatest monomial factor is 1 and which can't be written as a product of polynomials of lower degree. For example, $a^2 - 10a - 14$ is prime.

Example 1 Factor $x^2 + 6x + 8$.

Solution

1. The coefficient of the linear term is positive.

The pattern is $(x + ?)(x + ?)$.

List the positive factors of 8.

Factors of 8		Sum of the factors
1	8	9
2	4	6 ←

2. Find the pair of factors whose sum is 6: 4 and 2.

3. Therefore $x^2 + 6x + 8 = (x + 4)(x + 2)$.

You can check the result by multiplying $(x + 4)$ and $(x + 2)$.

$$(x + 4)(x + 2) = x^2 + 2x + 4x + 8 = x^2 + 6x + 8 \checkmark$$

Example 2 Factor $x^2 - 8x + 15$.

Solution

1. The coefficient of the linear term is negative.

The pattern is $(x - ?)(x - ?)$

List the pairs of negative factors of 15.

Factors of 15		Sum of the factors
-1	-15	-16
-3	-5	-8 ←

2. Find the pair of factors whose sum is -8 : -3 and -5 .

3. Therefore $x^2 - 8x + 15 = (x - 3)(x - 5)$.

Factor. Check by multiplying the factors. If the polynomial is not factorable, write *prime*.

1. $x^2 + 4x + 3$

2. $x^2 + 8x + 7$

3. $c^2 - 9c + 14$

4. $y^2 - 8y + 12$

5. $r^2 - 5r + 6$

6. $p^2 - 13p + 12$

7. $q^2 + 15q + 14$

8. $n^2 + 9n + 14$

9. $a^2 - 13a + 22$

10. $s^2 - 12s + 30$

11. $x^2 + 18x + 32$

12. $x^2 - 15x + 26$

5-7 Factoring Pattern for $x^2 + bx + c$, c positive (continued)**Example 3** Factor $y^2 - 10y + 16$.

- Solution**
1. Since -10 is negative, think of the negative factors of 16 in your head.
(After a little practice you will not need to write all the factors down.)
 2. Select the factors of 16 with sum -10 : -2 and -8 .
 3. Therefore $y^2 - 10y + 16 = (y - 2)(y - 8)$.

Factor. Check by multiplying the factors. If the polynomial is not factorable, write *prime*.

13. $a^2 + 10a + 30$

14. $x^2 - 19x + 60$

15. $k^2 - 21k + 54$

16. $n^2 + 23n + 90$

17. $k^2 - 10k + 21$

18. $x^2 - 14x + 45$

19. $k^2 + 7k + 12$

20. $x^2 - 16x + 48$

21. $a^2 - 11a + 20$

22. $x^2 + 22x + 72$

23. $72 - 17z + z^2$

24. $20 - 12c + c^2$

25. $54 - 15a + a^2$

26. $63 - 16c + c^2$

Example 4 Factor $x^2 - 12xy + 32y^2$.

- Solution** $x^2 - 12xy + 32y^2 = (x - ?)(x - ?)$ Write the factoring pattern.
 $= (x - 4y)(x - 8y)$ Fill in the negative factors of $32y^2$.

Factor. Check by multiplying the factors. If the polynomial is not factorable, write *prime*.

27. $x^2 - 11xy + 28y^2$

28. $a^2 - 9ab + 18b^2$

29. $c^2 - 18cd + 45d^2$

30. $x^2 - 10xy + 21y^2$

31. $c^2 - 14cd + 24d^2$

32. $x^2 + 11xy + 30y^2$

33. $y^2 - 16yz + 48z^2$

34. $a^2 - 18ab + 45b^2$

35. $d^2 + 10de + 24e^2$

36. $y^2 - 27yz + 72z^2$

Mixed Review Exercises

Solve.

1. $-12 + x = -7$

2. $d + (-4) = -9$

3. $-12 + b = 13$

4. $a + 3 = |2 - 9|$

5. $17m = 68$

6. $3p + 15 = -60$

7. $-\frac{1}{3}x = 9$

8. $\frac{r}{2} - 3 = 6$

9. $-18x = 162$