$\boxed{21\ p377}$ Use the definition of area to find an expression for the area under the graph of

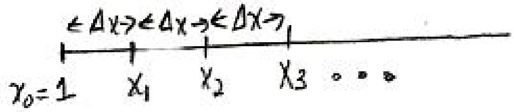
$$f(x) = \frac{2x}{x^2+1}$$

for $1 \le x \le 3$.

Soln. Partition $[1,3]$ into n subintervals of length

$$\Delta x = \frac{3-1}{n} = \frac{2}{n}.$$

The left- and right endpoints are



$x_0 = 1$
$x_1 = 1 + \Delta x = 1 + \frac{2}{n}$
$x_2 = 1 + 2\Delta x = 1 + 2\left(\frac{2}{n}\right)$
$\vdots$

$$\boxed{x_i = 1 + i\,\Delta x = 1 + i\left(\frac{2}{n}\right) \quad \text{for } i = 0, 1, 2, \ldots n.}$$

Note $x_n = 1 + n\left(\frac{2}{n}\right) = 1 + 2 = 3$.

Evaluating at the right endpts, the corresponding y-values are

$$y_i = f(x_i) = \frac{2x_i}{x_i^2+1} = \frac{2\left(1+\frac{2i}{n}\right)}{\left(1+\frac{2i}{n}\right)^2+1}$$

So, $$R_n = \sum_{i=1}^{n} f(x_i)\,\Delta x = \sum_{i=1}^{n} \frac{2\left(1+\frac{2i}{n}\right)}{\left(1+\frac{2i}{n}\right)^2+1} \cdot \frac{2}{n}.$$

$$\boxed{\text{Area} = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{2\left(1+\frac{2i}{n}\right)}{\left(1+\frac{2i}{n}\right)^2+1} \cdot \frac{2}{n}}$$ ⇐ Answer

$\boxed{\text{Remark.}}$ Taking $n = 200$,

$$\text{Area} \approx 0.01 \sum_{i=1}^{200} \frac{2(1+.01i)}{(1+.01i)^2+1} = 1.607436.$$

Exact answer $= \int_1^3 \frac{2x}{x^2+1}\,dx \quad u = x^2+1$

$= \int_2^{10} \frac{du}{u} = \ln 10 - \ln 2$
$= \ln 5 = 1.6094\ldots$