

### 3-5 Equations with the Variable on Both Sides

**Objective:** To solve equations with the variable on both sides.

**Example 1** Solve  $5x = 2x + 15$ .

**Solution**  $5x - 2x = 2x + 15 - 2x$  Subtract  $2x$  from each side.      *Check:*  $5(5) \stackrel{?}{=} 2(5) + 15$   
 $3x = 15$        $25 \stackrel{?}{=} 10 + 15$   
 $x = 5$        $25 = 25 \checkmark$

The solution set is  $\{5\}$ .

**Example 2** Solve  $4x = 30 - x$ .

**Solution**  $4x + x = 30 - x + x$  Add  $x$  to each side.  
 $5x = 30$   
 $x = 6$       The solution set is  $\{6\}$ .

Solve.

1.  $5n = 3n + 8$
2.  $7a = 2a + 30$
3.  $y = 20 - 3y$
4.  $3b = 80 - 5b$
5.  $10n = 36 - 2n$
6.  $2x = 20 - 8x$
7.  $21a = 56 + 7a$
8.  $30 + 6x = 12x$
9.  $-9a = -12a - 45$
10.  $33c + 60 = 21c$
11.  $72 - 4n = -22n$
12.  $-11a = -12a - 21$

**Example 3** Solve  $\frac{2}{5}x + 12 = x$ .

**Solution**  $\frac{2}{5}x + 12 - \frac{2}{5}x = x - \frac{2}{5}x$  Subtract  $\frac{2}{5}x$  from each side.  
 $12 = \frac{5}{5}x - \frac{2}{5}x$  Rewrite  $1x$  as  $\frac{5}{5}x$ .  
 $12 = \frac{3}{5}x$   
 $\frac{5}{3} \cdot \frac{12}{1} = \frac{5}{3} \left( \frac{3}{5}x \right)$  Multiply each side by  $\frac{5}{3}$ , the reciprocal of  $\frac{3}{5}$ .  
 $20 = x$       The solution set is  $\{20\}$ .

**Example 4** Solve  $\frac{6+x}{3} = x$ .

**Solution**  $3\left(\frac{6+x}{3}\right) = 3 \cdot x$  Multiply each side by 3, the reciprocal of  $\frac{1}{3}$ .  
 $6 + x = 3x$   
 $6 + x - x = 3x - x$  Subtract  $x$  from each side.  
 $6 = 2x$   
 $3 = x$       The solution set is  $\{3\}$ .

**3-5 Equations with the Variable on Both Sides (continued)**

Solve.

13.  $\frac{2}{3}x - 5 = x$       14.  $\frac{3}{4}x - 8 = x$       15.  $x = \frac{1}{2}x + 7$       16.  $x = \frac{4}{5}x - 9$   
 17.  $\frac{x-2}{3} = x$       18.  $\frac{3+y}{4} = y$       19.  $y = \frac{7-2y}{5}$       20.  $x = \frac{9+x}{4}$

**Vocabulary**

**Empty set or null set** The set with no members.

**Identity** An equation that is true for every value of the variable(s).

**Symbol**  $\phi$  (empty set, or the null set)

**CAUTION** An equation may have no solution, or it may be satisfied by every real number.

**Example 5** Solve:    a.  $5(a - 2) - 3 = 3a + 7 + 2a$     b.  $\frac{1}{3}(24x - 15) = 8x - 5$

**Solution**    a.  $5a - 10 - 3 = 5a + 7$   
                    $5a - 13 = 5a + 7$   
                    $-13 = 7$  ← **False**

The equation has *no solution*.

b.  $8x - 5 = 8x - 5$  ← **Identity**  
 An identity is true for every value of the variable.

The solution set is {real numbers}.

Solve each equation. If the equation is an identity or if it has no solution, write *identity* or *no solution*.

21.  $2(x - 3) = 5x$       22.  $4(y - 5) = 9y$       23.  $3n = 6(3 - n)$   
 24.  $-3m = 5(2 - m)$       25.  $2(a - 1) = 2a + 3$       26.  $\frac{1}{4}(28x - 8) = 7x - 2$   
 27.  $\frac{1}{3}(3x - 3) + 2 = 2x$       28.  $4(a - 1) - 5 = 3a + 7$       29.  $3(5 + y) - y = 2y + 15$   
 30.  $4a + 7 + a = 3(a - 1)$       31.  $\frac{3n - 15}{4} = 2n$       32.  $\frac{2n - 9}{2} = n$

**Mixed Review Exercises**

Simplify.

1.  $3 + \left(-\frac{1}{3}\right) + \left(-\frac{5}{3}\right)$       2.  $-2\frac{3}{4} + 1\frac{1}{4}$       3.  $-115 - (-10)$   
 4.  $15x + (-3x) - 2$       5.  $-4y + 5 + 18y + 23$       6.  $6(-2)(-5)(-4)$

Solve.

7.  $-2 - x = 5$       8.  $4 + (1 + k) = 2$       9.  $3x = -276$   
 10.  $\frac{1}{2}x = 3\frac{1}{2}$       11.  $\frac{x}{6} = 7$       12.  $-10\frac{2}{3} = -\frac{1}{3}x$