Section 4.3 Problems

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Class example. Find the local minimum and maximum values and inflection points (if any) of $f(x) = x^3 + 2x^2 + 1$.

> $f := x \to x^3 + 2x^2 + 1.$

$$f := x \to x^3 + 2x^2 + 1. \qquad (1)$$

>

Solution. The following details were worked out in class.

Local minimum value is 1 at $x = 0$. Local maximum value is $\frac{59}{27} \approx 2.2$ at $x = -\frac{4}{3}$.

Inflection point is $\left(-\frac{2}{3}, \frac{43}{27}\right)$ where $\frac{43}{27} \approx 1.6$.

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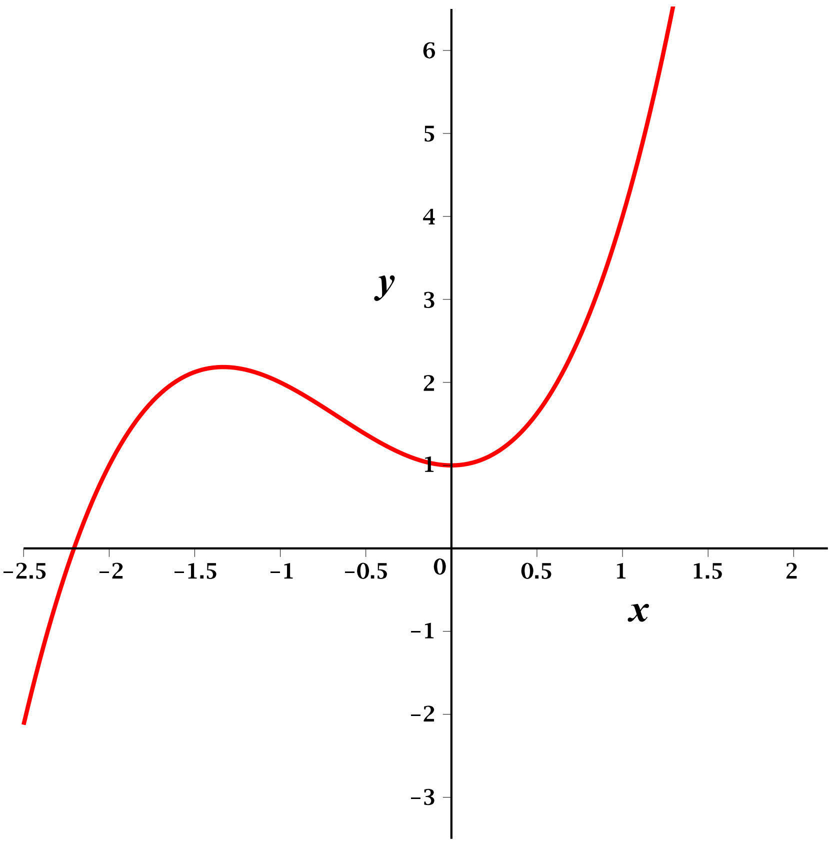
The following is the graph of the function:

> $plot(f(x), x = -2.5..2.2, y = -3.5..6.5, tickmarks = ([spacing(0.5), spacing(1)]), thickness = 2, font = [Times, bold, 8], labelfont = [Roman, bold, 14], color = red)$



y
x
6
5
4
3
2
1
0
−1
−2
−3
−2.5
−2
−1.5
−1
−0.5
0.5
1
1.5
2

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#12, p. 301. Find the local minimum and maximum values and inflection points (if any) of $f(x) = \dfrac{x}{x^2+1}$.

> $f := x \rightarrow \dfrac{x}{x^2+1}$

$$f := x \rightarrow \frac{x}{x^2+1} \qquad \textbf{(2)}$$

Solution. The following details were worked out in class.

Local minimum value is $-\dfrac{1}{2}$ at $x = -1$. Local maximum value is $\dfrac{1}{2}$ at $x = 1$.

Inflection points are : $\left(-\sqrt{3}, -\dfrac{\sqrt{3}}{4}\right)$, $(0, 0)$, and $\left(\sqrt{3}, \dfrac{\sqrt{3}}{4}\right)$ where

> $\sqrt{3}, evalf\left(\sqrt{3}, 5\right), \dfrac{\sqrt{3}}{4}, evalf\left(\dfrac{\sqrt{3}}{4}, 3\right)$

$$\sqrt{3},\ 1.7321,\ \frac{1}{4}\sqrt{3},\ 0.432 \qquad \textbf{(3)}$$

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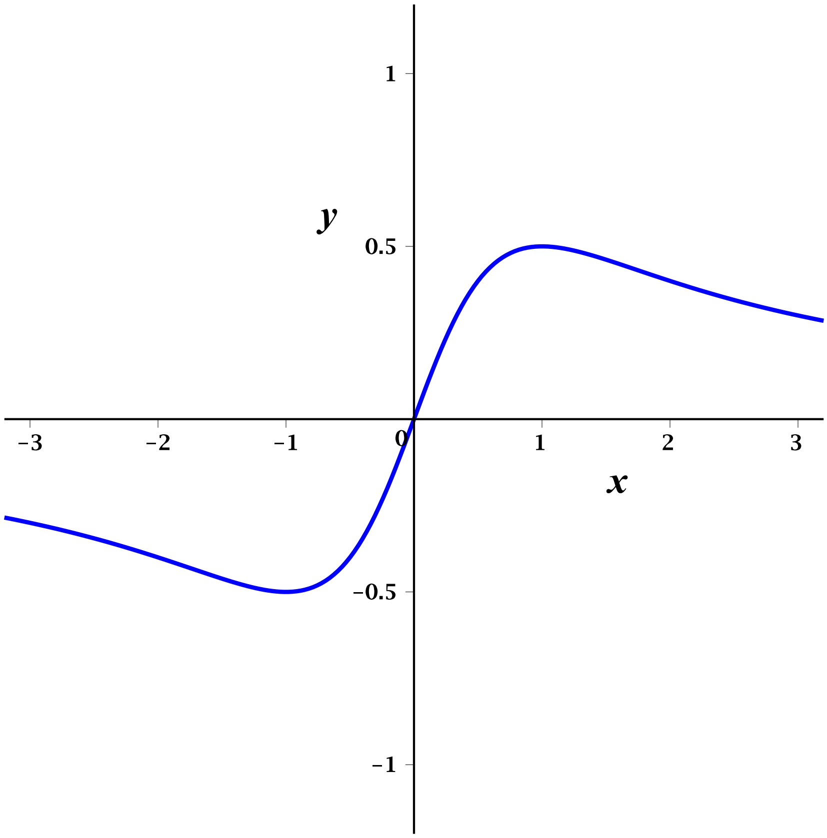
The following is the graph of the function:

> $plot(f(x), x=-3.2..3.2, y=-1.2..1.2, tickmarks=([spacing(1), spacing(0.5)]),$
> $thickness=2, font=[Times, bold, 8], labelfont=[Roman, bold, 14], color=blue)$



y
1
0.5
−0.5
−1
x
−3
−2
−1
0
1
2
3

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Application (#7, p. 337 in Section 4.7).
Find the dimensions of a rectangle with perimeter 100 meters whose area is as large as possible.

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Application.
A three-sided fence is to be built next to a straight section of a river, which forms the fourth side of a rectangle.
The enclosed area is to equal 1800 square feet. Find the minimum perimeter and dimensions of the rectangle.

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Application (#2, p. 337 in Section 4.7).
Find two numbers whose difference 100 and whose product is a minimum.

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Application (#15, p. 337 in Section 4.7).
If 1200 square centimeters of material is available to make a box with a square base and an open top, find the largest possible volume of the box.

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