# Section 4.1

>

A ***critical number*** of a function $f$ is a number $c$ in the domain of $f$ such that either $f'(c) = 0$ or $f'(c)$ does not exist.

>

**#30, p. 283.** Find the critical numbers of the function $f(x) = x^3 + 6x^2 - 15x$.

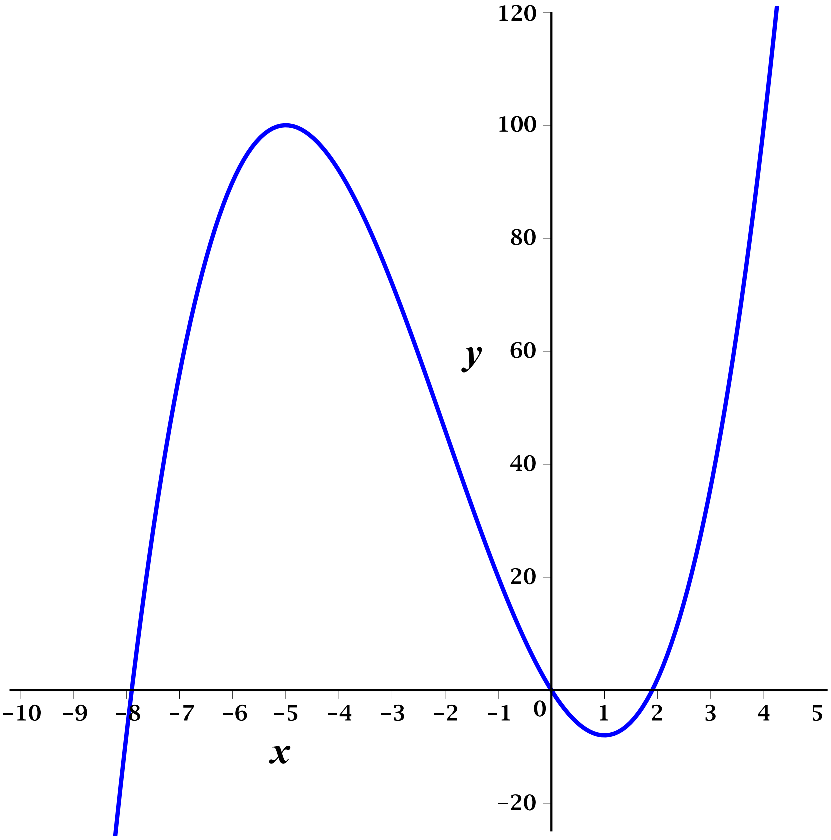
> $f := x \to x^3 + 6x^2 - 15\,x$

$$f := x \to x^3 + 6\,x^2 - 15\,x \qquad (1)$$

**Solution.** The critical numbers are $x = -5$ and $x = 1$.

>

> $plot(f(x), x = -10.2\,..5.2, y = -25\,..120, tickmarks = ([spacing(1), spacing(20)]),$
$thickness = 2, font = [Times, bold, 8], labelfont = [Roman, bold, 14], color = blue)$

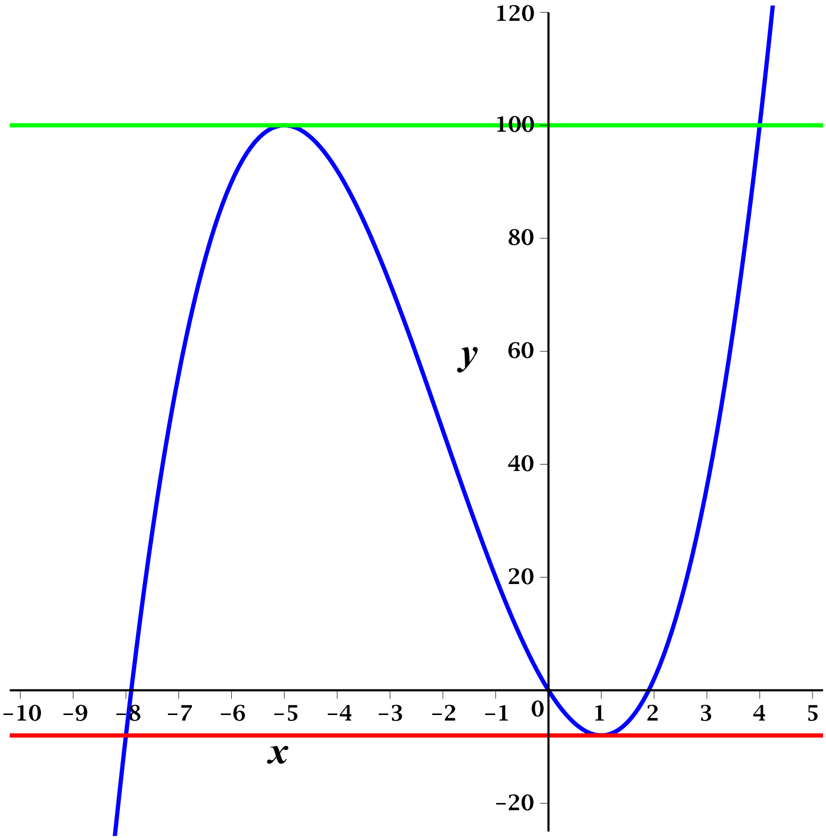



>

Equations of the tangent line to the graph at $x=1$ is $y=f(1)=-8$. And at $x=-5$, $y=100$.

>

> $plot([f(x), -8, 100], x=-10.2..5.2, y=-25..120, tickmarks=([spacing(1), spacing(20)]), thickness=2, font=[Times, bold, 8], labelfont=[Roman, bold, 14], color=[blue, red, green])$




>

**#44, p. 284.** Find the critical numbers of the function $f(x) = x^{-2} \ln(x)$.

>

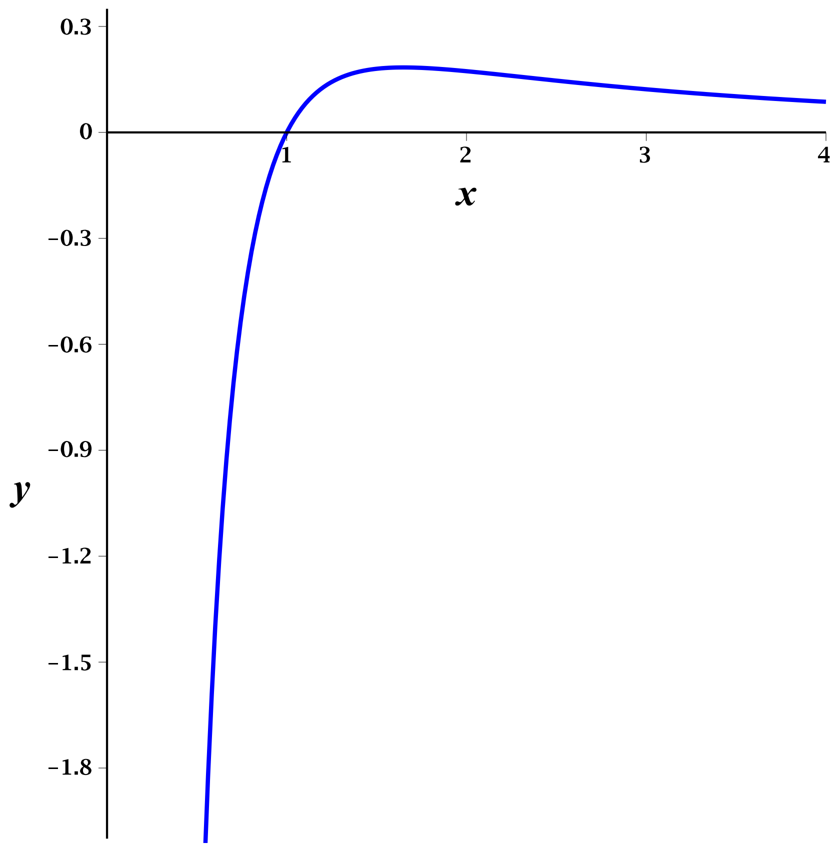
> $f := x \rightarrow x^{-2} \cdot \ln(x)$

$$f := x \rightarrow \frac{\ln(x)}{x^2} \tag{2}$$

**Solution.** The only critical number is $x = \sqrt{e}$.

>

> $plot(f(x), x = 0..4, y = -2..0.35, tickmarks = ([spacing(1), spacing(0.3)]), thickness = 2, font = [Times, bold, 8], labelfont = [Roman, bold, 14], color = blue)$

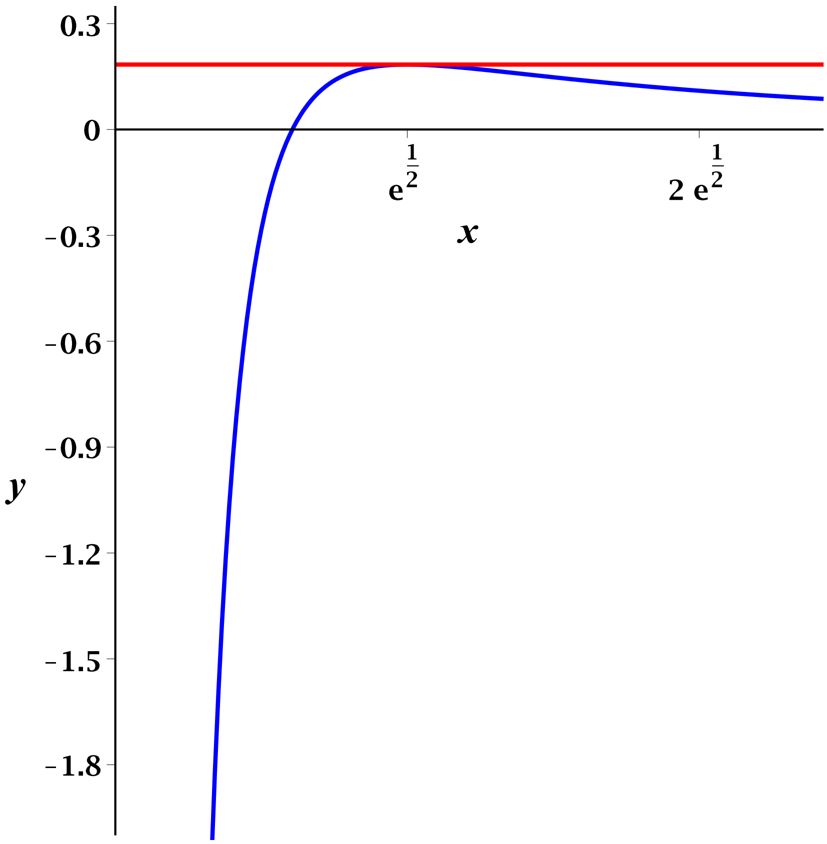



>

Equation of the tangent line to the graph at $x = \sqrt{e}$ is $y = \frac{1}{2e}$.

>

> $plot\left(\left[f(x), \frac{1}{2e^1}\right], x=0..4, y=-2..0.35, tickmarks=\left(\left[spacing\left(\sqrt{e^1}\right), spacing(0.3)\right]\right),\right.$ $thickness=2, font=[Times, bold, 10], labelfont=[Roman, bold, 14], color=[blue, red]\big)$




>

**#38, p. 283.** Find the critical numbers of the function $g(x) = \sqrt[3]{4 - x^2}$.

>

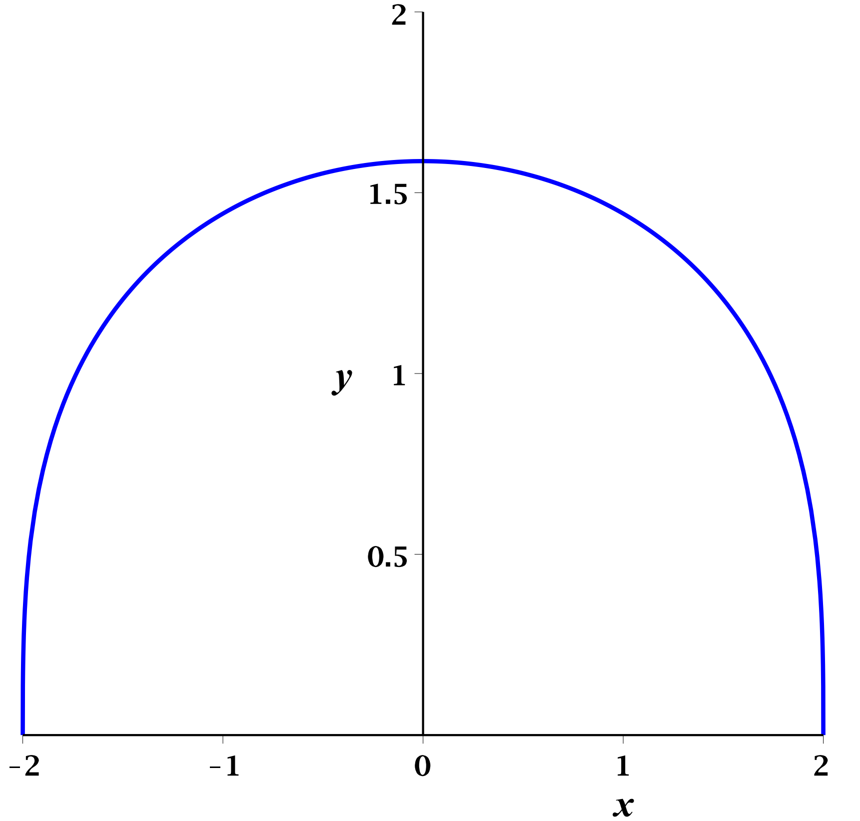
> $g := x \rightarrow \sqrt[3]{4 - x^2}$

$$g := x \rightarrow (4 - x^2)^{1/3} \tag{3}$$

**Solution.** *The critical numbers are* $x = -2, 0, 2$.

>

> *plot*(*g*(*x*), *x* =-2 ..2, *y* = 0 ..2, *tickmarks* = ([*spacing*(1), *spacing*(0.5)]), *thickness* = 2, *font* = [*Times*, *bold*, 10], *labelfont* = [*Roman*, *bold*, 14], *color* = *blue*)




>

>

**Theorem 7 on p. 280.** If a function $f$ has a local maximum or minimum at a number $c$, then $c$ is a critical number of $f$.

>

It follows from Theorem 7 and the *Extreme Value Theorem* (see p. 278) that the absolute extremum of a continuous function on a closed interval occurs at either a critical number or at an endpoint of the interval.

**#48, p. 284.** Find the absolute extrema values of $f(x) = 5 + 54x - 2x^3$ on [0, 4].

>

**Solution.** The absolute maximum value is 113 at $x = 3$. The absolute minimum value is 5 at $x = 0$.

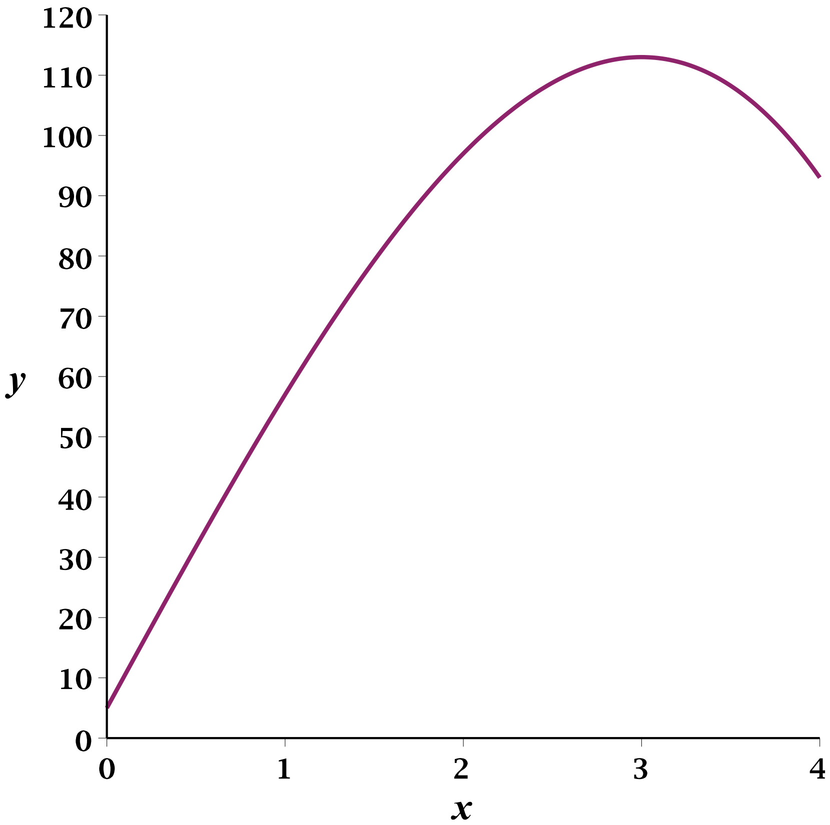
>

> $f := x \to 5 + 54\,x - 2\,x^3$

$$f := x \to 5 + 54\,x - 2\,x^3 \tag{4}$$

>

> $plot(f(x), x = 0..4, y = 0..120, tickmarks = ([spacing(1), spacing(10)]), thickness = 2, font = [Times, bold, 10], labelfont = [Roman, bold, 14], color = maroon)$


120
110
100
90
80
70
60
50
40
30
20
10
0
y
0
1
2
3
4
x
>

**#60, p. 284.** Find the absolute extrema values of $f(x) = x \cdot e^{\frac{x}{2}}$ on [-3,1].

>

**Solution.** The absolute maximum value is $\sqrt{e} \approx 1.6487$ at $x = 1$. The absolute minimum value is $-2e^{-1} \approx -0.73575$ at $x = -2$.

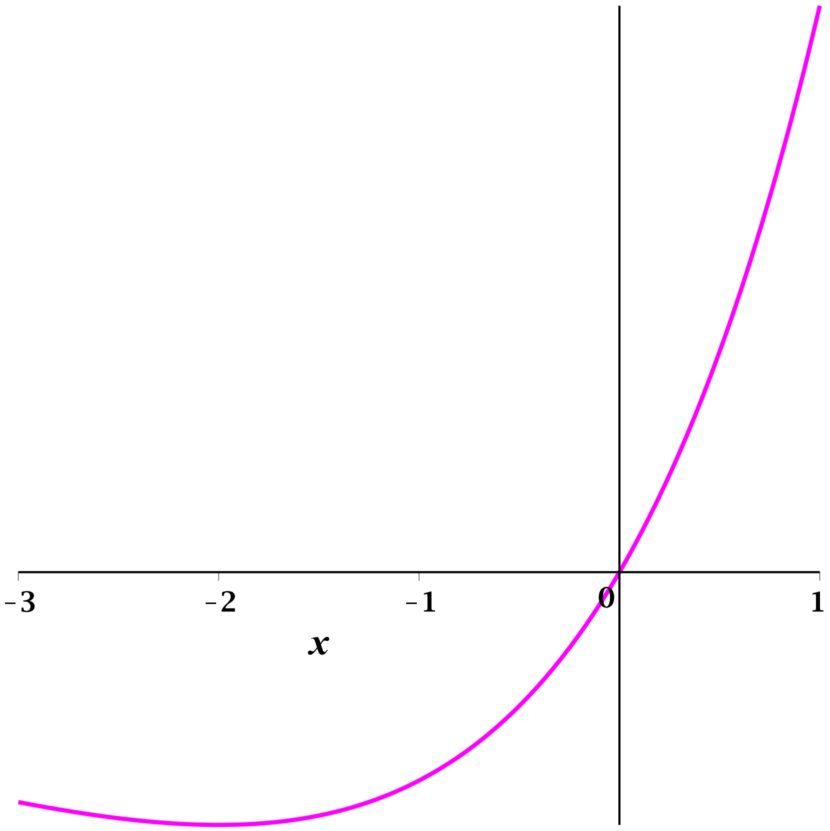
>

> $f := x \rightarrow x \cdot e^{\frac{x}{2}}$

$$f := x \rightarrow x\, e^{\frac{1}{2} x} \qquad (5)$$

>

> $plot(f(x), x = -3..1, tickmarks = ([spacing(1), spacing(10)]), thickness = 2, font = [Times, bold, 10], labelfont = [Roman, bold, 14], color = magenta)$



**Application (#7, p. 337 in Section 4.7).**

Find the dimensions of a rectangle with perimeter 100 meters whose area is as large as possible.

>

>