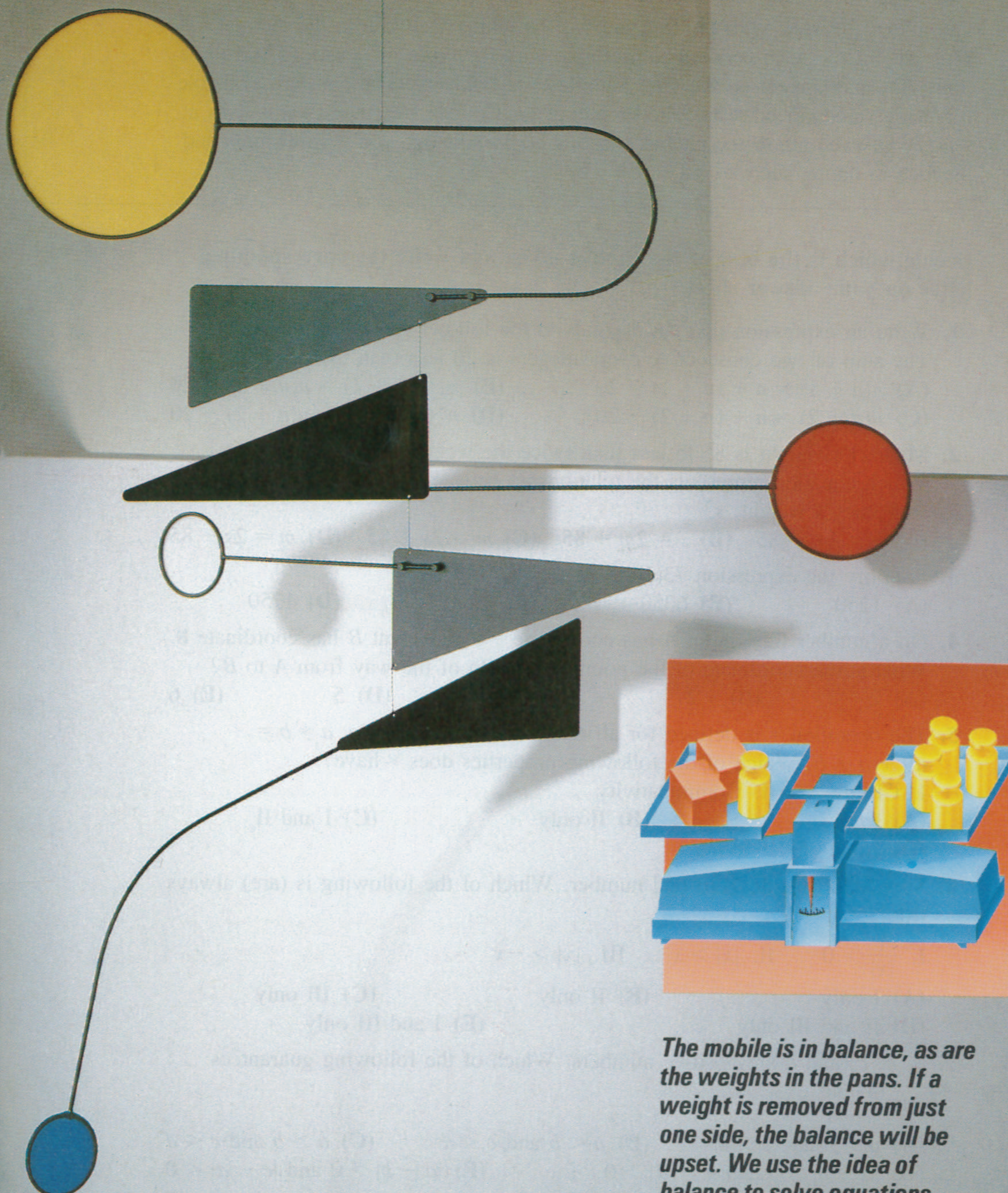
3 Solving Equations and Problems



The mobile is in balance, as are the weights in the pans. If a weight is removed from just one side, the balance will be upset. We use the idea of balance to solve equations.

Transforming Equations

3-1 Transforming Equations: Addition and Subtraction

Objective To solve equations using addition or subtraction.

Two soccer teams are tied at half time: 2 to 2. If each team scores 3 goals in the next half, then the score will still be tied:

$$2 + 3 = 2 + 3$$

Two sporting goods stores charge \$36 for a soccer ball. If, during a spring sale, each store reduces the price by \$5, both stores will still be charging the same price:

$$36 - 5 = 36 - 5$$

The examples above illustrate the following properties of equality.

Addition Property of Equality

If a, b, and c are any real numbers, and $a = b$, then

$$\boldsymbol{a + c = b + c} \qquad \text{and} \qquad \boldsymbol{c + a = c + b.}$$

If the same number is added to equal numbers, the sums are equal.

Subtraction Property of Equality

If a, b, and c are any real numbers, and $a = b$, then

$$\boldsymbol{a - c = b - c.}$$

If the same number is subtracted from equal numbers, the differences are equal.

Notice that the subtraction property of equality is just a special case of the addition property, since subtracting the number c is the same as adding $-c$. The addition property of equality guarantees that if $a = b$,

$$a + (-c) = b + (-c)$$

$$\text{or} \qquad a - c = b - c.$$

Examples 1 and 2 show how to use the addition and subtraction properties of equality to solve some equations. You add the same number to, or subtract the same number from, each side of the given equation in order to get an equation with the variable alone on one side.

Example 1 Solve $x - 8 = 17$.

Solution

$$x - 8 = 17 \quad \{\text{Copy the equation.}$$
$$x - 8 + 8 = 17 + 8$$
$$x = 25$$
Add 8 to each side and then simplify.

Because errors may occur in solving equations, you should check that each solution of the final equation satisfies the *original equation*.

Check: $x - 8 = 17 \leftarrow$ original equation
$25 - 8 \stackrel{?}{=} 17$
$17 = 17$ ✓ $\quad \therefore$ the solution set is $\{25\}$. ***Answer***

The properties of real numbers guarantee in Example 1 that if the original equation, $x - 8 = 17$, is true for some value of x, then the final equation, $x = 25$, is also true for that value of x, and vice versa. Therefore the two equations have the same solution set, $\{25\}$.

Example 2 Solve $-5 = n + 13$.

Solution

$$-5 = n + 13$$
$$-5 - 13 = n + 13 - 13$$
$$-18 = n$$
Subtract 13 from each side and then simplify.

Check: $-5 = n + 13$
$-5 \stackrel{?}{=} -18 + 13$
$-5 = -5$ ✓ $\quad \therefore$ the solution set is $\{-18\}$. ***Answer***

Equations having the same solution set over a given domain are called **equivalent equations** over that domain. In Example 1, the equations $x - 8 = 17$ and $x = 25$ are equivalent equations. In Example 2, the equations $-5 = n + 13$ and $-18 = n$ are equivalent equations.

It is often possible to change, or *transform*, an equation into a simpler equivalent equation by using substitution or the addition and subtraction properties. The goal is to obtain a simpler equation whose solution or solutions can be easily seen.

Transforming an Equation into an Equivalent Equation

Transformation by Substitution
Substitute an equivalent expression for any expression in a given equation.

Transformation by Addition
Add the same real number to each side of a given equation.

Transformation by Subtraction
Subtract the same real number from each side of a given equation.

Oral Exercises

Describe how to change each equation to produce an equivalent equation with the variable alone on one side. Then state this equivalent equation.

Sample 1 $x - 3 = 5$ **Solution** Add 3 to each side; $x = 8$

Sample 2 $z + 5 = 4$ **Solution** Subtract 5 from each side; $z = -1$

1. $x + 5 = 9$ **2.** $x + 3 = 8$ **3.** $t - 2 = 7$

4. $w - 11 = 4$ **5.** $a + 9 = 1$ **6.** $b + 7 = 6$

7. $6 + n = 0$ **8.** $-4 + y = 0$ **9.** $-5 + m = 5$

10. $-1 + r = -1$ **11.** $-8 + t = -8$ **12.** $-5 = u + 9$

13. $-5 = t - 9$ **14.** $4 = -2 + s$ **15.** $-1 = 5 + k$

16. $z - \frac{1}{4} = \frac{1}{4}$ **17.** $\frac{4}{5} = \frac{1}{5} + d$ **18.** $h + 1.7 = -2.1$

19. $x + 3.2 = 4.5$ **20.** $-4.2 = z + 2.1$ **21.** $-1 + x = \frac{1}{3}$

Written Exercises

Solve.

A

1. $x - 7 = 13$ **2.** $y - 9 = 17$ **3.** $z + 8 = 31$

4. $x + 15 = 27$ **5.** $-52 + m = 84$ **6.** $-49 + n = 63$

7. $t - 25 = -18$ **8.** $x - 26 = 18$ **9.** $p + 18 = -32$

10. $y + 32 = -45$ **11.** $0 = 38 + k$ **12.** $0 = z - 14$

13. $-19 + a = 23$ **14.** $-32 + b = 82$ **15.** $c + 9 = 5$

16. $x - 8 = 25$ **17.** $f + 7 = 9 - 2$ **18.** $g - 6 = 14 - 8$

19. $z - 57 = -67$ **20.** $x - 97 = -105$ **21.** $-0.7 + k = -1.7$

22. $-1.8 + h = -3.8$ **23.** $4.5 = x + 1.6$ **24.** $3.9 = y + 1.2$

Sample 1 $-x + 7 = 2$

Solution

$$-x + 7 - 7 = 2 - 7$$
$$-x = -5$$
$$x = 5$$

{Remember that the opposite of $-x$ is x and the opposite of -5 is 5.

$\therefore$ the solution set is $\{5\}$. ***Answer***

B

25. $-x + 6 = 4$ **26.** $-y + 5 = 17$ **27.** $21 - x = 28$

28. $9 - y = 16$ **29.** $8 = -x + 18$ **30.** $11 = 32 - y$

Solve.

31. $-8 - y = 9$
32. $7 = -12 + e$
33. $13 = -y + 8$
34. $(c + 2) + 8 = 4$
35. $(r + 4) + 2 = 1$
36. $2 = 10 + (x - 2)$
37. $8 = 16 + (y - 1)$
38. $-2 + (1 + p) = 5$
39. $-3 + (1 + n) = 9$
40. $(a - 3) + 19 = 125$
41. $(b - 6) + 14 = 100$
42. $4 - (1 + x) = 5$
43. $2 - (3 + y) = 6$
44. $1 = -2 - (4 - w)$
45. $11 = 7 - (1 - q)$

Sample 2 $|x| + 4 = 13$

Solution $|x| + 4 - 4 = 13 - 4$

$|x| = 9$

$x = 9$ or $x = -9$ $\quad \therefore$ the solution set is $\{9, -9\}$ ***Answer***

C

46. $|y| - 2 = 8$
47. $|z| + 10 = 28$
48. $-7 + |s| = 0$
49. $6 + |t| = 14$
50. $|x| + (-2) = 4$
51. $|y| + (-1) = 1$
52. $0 = -5 + |r|$
53. $2 = 6 + |t|$
54. $-(|a| - 9) = 1$
55. $-(|x| + 2) = -6$
56. $4 - (2 - |n|) = 2$
57. $7 - (3 - |m|) = 8$
58. $9 - (|s| + 7) = 4$
59. $-3 + (15 - |a|) = 12$
60. $(|e| - |-8|) + 15 = 7$

Problems

Write an equation based on the facts of the problem. Then solve the equation and answer the question asked in the problem.

Sample 1 37 less than a number is -19. What is the number?

Solution $n - 37 = -19$

$n = 18$ $\quad \therefore$ the number is 18. ***Answer***

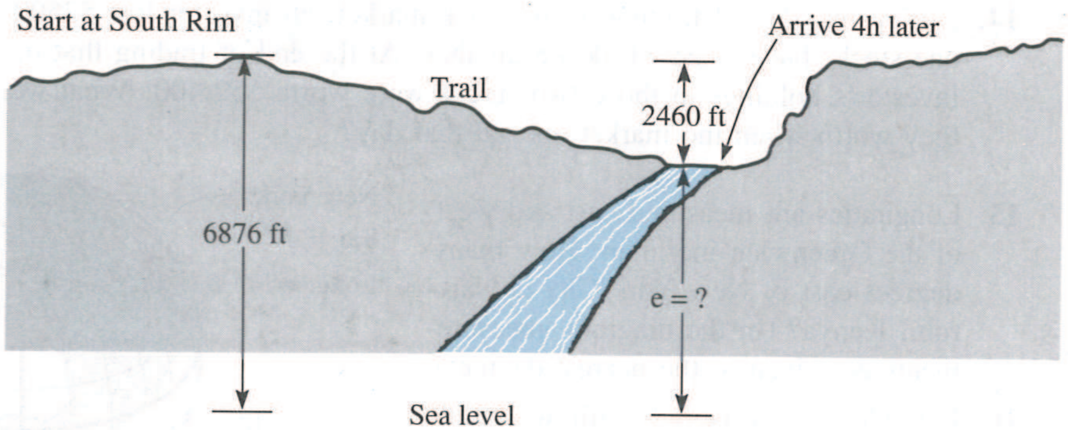
A

1. Fifty-one more than a number is -12. What is the number?
2. Twenty-two less than a number is -7. What is the number?
3. If a number is increased by 28, the result is 7. What is the number?
4. If a number is decreased by 8, the result is -21. What is the number?
5. If -8 is subtracted from a number, the result is 84. What is the number?
6. If -15 is subtracted from a number, the result is -29. What is the number?

Sample 2 Wylie hiked into the Grand Canyon from its South Rim, which is 6876 ft above sea level. Walking along the 7.8 mi Bright Angel Trail, he reached the Colorado River in 4 h. At that point he was 2460 ft lower in the Canyon than at his starting point. How far above sea level is the Colorado River at this point?

Solution

Step 1 You are asked to find the river's elevation above sea level at the point where it crosses the Bright Angel Trail. Make a sketch to show the given information.



Step 2 Let e = the elevation of the river.

Step 3 $e + 2460 = 6876$

Step 4 $e + 2460 - 2460 = 6876 - 2460$

$e = 4416$

Step 5 *Check:* The check is left to you.

$\therefore$ the Colorado River is 4416 ft above sea level at the point where it meets the trail. ***Answer***

Notice that two of the given facts were not used in solving the problem: the length of the trail and the time spent hiking.

7. A lion can run 18 mi/h faster than a giraffe. If a lion can run 50 mi/h, how fast can a giraffe run?

8. Corita ran the 400-meter dash in 56.8 s. This was 1.3 s less than her previous time. What was her previous time?

9. The desert temperature rose 25° C between 6 A.M. and noon. If the temperature at noon was 18° C, what was the temperature at 6 A.M.?

10. The temperature at the summit of Mt. Mansfield dropped 17° F between 4 P.M. and 11 P.M. If the temperature at 11 P.M.was −11° F, what was the temperature at 4 P.M.?

11. Enrico paid \$4.75 for a sandwich, a drink, and frozen yogurt. He remembered that the drink and the yogurt were each \$1.15 and that the sandwich had too much mustard, but he forgot the price of the sandwich. How much did the sandwich cost?

12. Ruth Panoyan had 45 sheets of graph paper. She gave five sheets to each of the six students she tutored and put the remaining sheets in her desk. How many did she put in her desk?

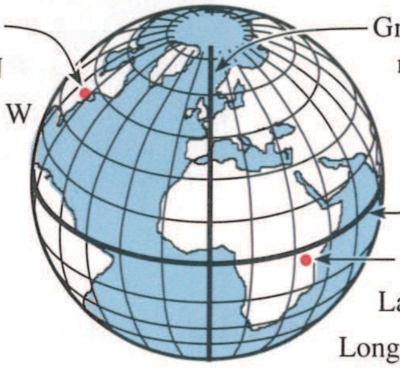
Write an equation based on the facts of the problem. Then solve the equation and answer the question asked in the problem.

B **13.** A factory hired 130 new workers during a year in which 27 workers retired and 59 left for other reasons. If there were 498 workers at the end of the year, how many were there at the beginning of the year?

14. During one day of trading in the stock market, an investor lost $2500 on one stock, but gained $1700 on another. At the end of trading that day, the investor's holdings in those two stocks were worth $52,400. What were they worth when the market opened that day?

15. Longitudes are measured east and west of the Greenwich meridian. How many degrees east of New York City is Nairobi, Kenya? (In the diagram, measurements are given to the nearest degree.)

16. Latitudes are measured north and south of the equator. How many degrees south of New York City is Nairobi?



17. Gino paid $3.23 for two tubes of toothpaste. He paid the regular price of $1.79 for one tube. However, he bought the other one for less because he used a discount coupon. How much was the coupon worth?

18. Kerry bought a picture frame on sale for $4.69. A week later, she returned to buy another frame. However, she had to pay the regular price for the second one. If the two frames cost Kerry $10.64, how much had the store reduced the price for the sale?

C **19.** At 8:00 P.M. a scavenger hunt started at the town hall. Team 1 drove 5 km east to the golf course, while Team 2 drove 12 km west to the beach. By the time Team 2 had found some seaweed at the beach, Team 1 had already found an orange golf ball and driven 3 km back toward town hall. How far apart were the two teams at this point?

20. After traveling 387 miles from Los Angeles to San Francisco, Rick noted that his car's odometer read exactly 27972. This number reads the same backwards as forwards. What is the next such number and how far will he have to drive to get it to appear on the odometer?

Mixed Review Exercises

Evaluate if $a = 4$, $b = -6$, $c = -7$, and $d = 5$.

1. $a - |c - b|$ **2.** $(|b| - a) - (|d| - a)$ **3.** $2|a| - (-a)$

4. $\frac{b - 2c}{b + 2d}$ **5.** $\frac{4d + c + 1}{abc}$ **6.** $\frac{3ab}{c + d}$

Simplify.

7. $(-6)(-7)(10)$
8. $(-9 \cdot 12) + (-9 \cdot 18)$
9. $148a \div (-37)$
10. $96\left(-\frac{1}{24}\right)\left(-\frac{1}{4}\right)$
11. $-\frac{12b}{5} \cdot (-5)$
12. $\frac{1}{7}(21a - 7b) - \frac{1}{3}(12b - 6a)$

Application / Car Loans

A car is an expensive purchase. Most people do not have the money to pay for a car with cash. Instead, they pay part of the cash price as a *down payment* and borrow the rest by taking out a loan. The car buyer must pay the lender the amount of money borrowed, which is called the *principal,* plus *interest*.

Example Miriam bought a \$3600 used car. She made a \$630 down payment and got a loan for 36 months with payments of \$95 per month.

a. Find the total amount paid for the car.

b. Find the amount of interest Miriam had to pay.

Solution

a. Total paid = down payment + total of the monthly payments

$= 630 + 36(95)$

$= 630 + 3420$

$= 4050$ ∴ the total paid was \$4050. ***Answer***

b. Interest = total paid − cash price

$= 4050 - 3600$

$= 450$ ∴ Miriam had to pay \$450 in interest. ***Answer***

Exercises

a. Find the total amount to be paid on each vehicle.
b. Find the amount of interest the buyer would be paying.

1. Sean bought a \$3000 used car by making a \$480 down payment and getting a loan for two years with payments of \$115 per month.
2. The Valley Fruit Stand got a three-year loan at \$154 per month to pay for a \$5500 used truck. The down payment was \$451.
3. A \$4500 station wagon was advertised in the Daily Gazette. The buyer paid \$498 down and made 48 monthly payments of \$95 per month.
4. Linda bought a used car with a cash price of \$10,000. She made a down payment of \$620 and paid \$220 per month for four years.

3-2 Transforming Equations: Multiplication and Division

Objective To solve equations using multiplication or division.

At a hardware store, small construction supplies are often sold by the pound rather than by the number of items.

Suppose a pound of roof nails costs the same as a pound of floor nails. You would expect to pay the same price for *two* pounds of roof nails as for *two* pounds of floor nails, and the same price for *one-half* pound of roof nails as for *one-half* pound of floor nails.

This is an example of the multiplication and division properties of equality.

Multiplication Property of Equality

If a, b, and c are any real numbers, and $a = b$, then

$$ca = cb \qquad \text{and} \qquad ac = bc.$$

If equal numbers are multiplied by the same number, the products are equal.

Division Property of Equality

If a and b are any real numbers, c is any nonzero real number, and $a = b$, then

$$\frac{a}{c} = \frac{b}{c}.$$

If equal numbers are divided by the same *nonzero* number, the quotients are equal.

These properties give you two more ways to transform an equation into an equivalent equation. The others that you have already studied are listed on page 96.

Transforming an Equation into an Equivalent Equation

Transformation by Multiplication:
Multiply each side of a given equation by the same *nonzero* real number.

Transformation by Division:
Divide each side of a given equation by the same *nonzero* real number.

Example 1 Solve $6x = 222$.

Solution

$$6x = 222$$
$$\frac{6x}{6} = \frac{222}{6}$$
$$x = 37$$

{To get x alone on one side, divide each side by 6 (or multiply by $\frac{1}{6}$, the reciprocal of 6).

Check:
$$6x = 222$$
$$6(37) \stackrel{?}{=} 222$$
$$222 = 222 \quad \checkmark$$

$\therefore$ the solution set is $\{37\}$. ***Answer***

Example 2 Solve $8 = -\frac{2}{3}t$.

Solution

$$8 = -\frac{2}{3}t$$
$$-\frac{3}{2}(8) = -\frac{3}{2}\left(-\frac{2}{3}t\right)$$
$$-12 = t$$

{To get t alone on one side, multiply each side by $-\frac{3}{2}$, the reciprocal of $-\frac{2}{3}$.

Check:
$$8 = -\frac{2}{3}t$$
$$8 \stackrel{?}{=} -\frac{2}{3}(-12)$$
$$8 = 8 \quad \checkmark$$

$\therefore$ the solution set is $\{-12\}$. ***Answer***

Example 3 Solve: **a.** $\frac{m}{3} = -5$ **b.** $\frac{1}{4}s = 6\frac{1}{4}$

Solution

a.
$$3\left(\frac{m}{3}\right) = 3(-5)$$
$$m = -15$$

$\therefore$ the solution set is $\{-15\}$. ***Answer***

b.
$$\frac{1}{4}s = \frac{25}{4}$$
$$4\left(\frac{1}{4}s\right) = 4\left(\frac{25}{4}\right)$$
$$s = 25$$

$\therefore$ the solution set is $\{25\}$. ***Answer***

You know that zero cannot be a divisor (page 84). Do you know why zero is not allowed as a multiplier in transforming an equation? Look at the following equations.

(1) $5z = 45$
(2) $0 \cdot 5z = 0 \cdot 45$
(3) $(0 \cdot 5)z = 0 \cdot 45$
(4) $0 \cdot z = 0$

Equation (1) had just one root, namely 9. Equation 4 is satisfied by *any* real number. Since they do not have the same solution set, Equations (1) and (4) are *not* equivalent (see page 96). *In transforming an equation, never multiply by zero.*

Oral Exercises

Describe how you could produce an equivalent equation with the variable alone on one side. Then state the equivalent equation.

1. $8x = 16$
2. $5y = 15$
3. $3a = -12$
4. $-8a = 32$
5. $\frac{1}{2}b = 4$
6. $\frac{1}{3}t = 7$
7. $-\frac{1}{10}r = 5$
8. $-\frac{9}{5}m = 9$
9. $5 = \frac{5}{3}y$
10. $-7 = -\frac{7}{2}x$
11. $0 = -4k$
12. $x \div 8 = -1$
13. $n \div (-5) = 4$
14. $\frac{d}{2} = -6$
15. $-4 = \frac{x}{3}$
16. $-11f = -88$
17. $-25p = -75$
18. $7 = -\frac{u}{2}$

Written Exercises

Solve.

A

1. $4x = 44$
2. $5y = 65$
3. $6y = -18$
4. $3t = -27$
5. $-8c = 72$
6. $-6p = 42$
7. $-12a = -36$
8. $-9z = -63$
9. $\frac{1}{2}x = 12$
10. $\frac{1}{3}y = 18$
11. $\frac{x}{5} = -7$
12. $\frac{c}{4} = -9$
13. $-\frac{1}{8}b = 8$
14. $-\frac{1}{5}t = 17$
15. $\frac{2}{3}x = 12$
16. $\frac{5}{2}y = 10$
17. $\frac{3}{4}d = -60$
18. $\frac{5}{8}c = -20$
19. $-\frac{2}{3}t = 22$
20. $-\frac{2}{11}p = 14$
21. $600 = -25x$
22. $-324 = -18c$
23. $-17d = 0$
24. $252 = -14y$
25. $99 = -\frac{11}{5}x$
26. $24 = -\frac{3}{7}t$
27. $-6 = \frac{x}{3}$
28. $21 = \frac{c}{7}$

B

29. $\frac{1}{3}y = 2\frac{1}{3}$
30. $\frac{1}{2}t = 3\frac{1}{2}$
31. $-1 = 2.5z$
32. $0 = -4.5y$
33. $\frac{2}{5}a = 6\frac{2}{5}$
34. $\frac{3}{2}b = -14\frac{1}{2}$
35. $-\frac{y}{3} = 3\frac{2}{3}$
36. $-\frac{x}{2} = 11\frac{1}{2}$
37. $-\frac{y}{12} = \frac{1}{4}$
38. $-\frac{n}{4} = -\frac{3}{2}$
39. $0 = -\frac{x}{5}$
40. $-\frac{5}{3}x = 10$

C

41. $2|x| = 18$
42. $3|y| = 33$
43. $-7|t| = 42$
44. $-32 = 8|k|$
45. $\frac{|x|}{5} = 2$
46. $\frac{|a|}{8} = 4$
47. $3 = \frac{|n|}{7}$
48. $6 = \frac{|m|}{2}$
49. $\frac{5|c|}{3} = 10$
50. $\frac{4}{7}|x| = 16$
51. $7 - \frac{3|a|}{2} = 1$
52. $10 - \frac{4}{3}|b| = -2$

Problems

Write an equation based on the facts of each problem. Then solve the equation and the problem.

A

1. Five times a number is −375. Find the number.
2. Negative nine times a number is −108. Find the number.
3. One third of a number is −7. Find the number.
4. Three quarters of a number is 21. Find the number.
5. One hundred twenty seniors are on the honor roll. This represents one third of the senior class. How many seniors are there?
6. Two hundred twenty-five students play a team sport at Lincoln High School. These students represent $\frac{3}{8}$ of the total student population. How many students attend the school?
7. The perimeter of a square parking lot is 784 m. How long is each side of the lot?
8. The distance around the United States Pentagon building is one mile. How long is each side? (*Hint:* A regular pentagon has five equal sides.)

9. Luis ate three of the eight pizza slices. He paid \$2.70 as his share of the cost. How much did the whole pizza cost?
10. A restaurant charges \$2.50 for one eighth of a quiche. At this rate, how much does the restaurant receive for the whole quiche?
11. The Eagles won three times as many games as they lost. They won 21 games. How many games did they lose?
12. Buena Vista High School has five times as many black-and-white monitors as color monitors. The school has 40 black-and-white monitors for computers. How many color monitors does the school have?
13. How many apples, averaging 0.2 kg each, are included in a 50 kg shipment of apples?
14. A 75-watt bulb consumes 0.075 kW · h (kilowatt-hours) of energy when it burns for one hour. How long was the bulb left burning if it consumed 3.3 kW · h of energy?

B

15. A hard-cover book sells for \$16.50. The same title in paperback sells for \$4.95. How many hard-cover books must a dealer sell to take in as much money as he/she does for 30 paperback copies?
16. A certain real-estate agent receives \$6 for every \$100 of a house's selling price. How much was a house sold for if the agent received \$10,725?

17. An employer said that each worker received \$24 in fringe benefits for every \$100 in wages. At this rate, what wages were earned by a worker whose fringe benefits were valued at \$5100?

18. One kilogram of sea water contains, on average, 35 g of salt. How many grams of sea water contain 4.2 g of salt?

C 19. Raul drove $\frac{1}{4}$ mi from Exit 27 to Exit 28 in 18 s. At what rate was he traveling in mi/s? At what rate was he driving in mi/h? (*Hint:* rate · time = distance)

20. A police helicopter clocked a truck over a stretch of highway $\frac{1}{5}$ mi long. The truck traveled the distance in 10 s. At what rate was the truck traveling in mi/s? At what rate was it traveling in mi/h?

Mixed Review Exercises

Evaluate if $a = 2$, $b = -3$, and $c = 4$.

1. $7a - 2b$
2. $(3a - 2b)c$
3. $|b| + |c| - (-c)$
4. $|a| - |b + c|$
5. $\frac{-(5ab)}{2c}$
6. $\frac{2 + a}{c}$

Simplify.

7. $7a + 6 + 9a$
8. $5n - 9 + 9$
9. $10p - p + 2$
10. $-4(m + 2)$
11. $(x + 7)8$
12. $3(2y - 5)$

Calculator Key-In

Use the division key on a calculator to find a decimal equal to each expression.

Sample 1 $\frac{-36}{8}$ **Solution** $\frac{-36}{8} = -36 \div 8 = -4.5$

1. $\frac{3}{4}$
2. $\frac{-5}{8}$
3. $\frac{7}{-35}$
4. $\frac{-3}{-20}$
5. $\frac{1}{40}$
6. $\frac{-11}{4}$
7. $\frac{12}{-50}$
8. $\frac{-7}{-8}$
9. $\frac{31}{32}$
10. $\frac{43}{-64}$

11–20. Use the multiplication and reciprocal keys on a calculator to find a decimal equal to each expression in Exercises 1–10. (See Sample 2.) Are your answers the same as before?

Sample 2 $\frac{-36}{8}$ **Solution** $\frac{-36}{8} = -36 \cdot \frac{1}{8} = -4.5$

3-3 Using Several Transformations

Objective To solve equations by using more than one transformation.

If you start with n, multiply by 5, and subtract 9, you get the expression $5n - 9$. If you start with $5n - 9$, add 9, and divide by 5, you're back to n.

$$n \xrightarrow{\times 5} 5n \xrightarrow{-9} 5n - 9$$

$$5n - 9 \xrightarrow{+9} 5n \xrightarrow{\div 5} n$$

The addition of 9 "undoes" the subtraction of 9. We call addition and subtraction **inverse operations.** The diagram also shows that division by 5 "undoes" multiplication by 5. Multiplication and division are also inverse operations.

For all real numbers a and b,

$$(a + b) - b = a \quad \text{and} \quad (a - b) + b = a.$$

For all real numbers a and all *nonzero* real numbers b,

$$(ab) \div b = a \quad \text{and} \quad (a \div b)b = a.$$

Example 1 Solve $5n - 9 = 71$.

Solution

Use inverse operations:

$5n - 9 = 71$

$5n - 9 + 9 = 71 + 9$ — To undo the subtraction of 9 from $5n$, add 9 to each side.

$5n = 80$

$\frac{5n}{5} = \frac{80}{5}$ — To undo the multiplication of n by 5, divide each side by 5.

$n = 16$

$\therefore$ the solution set is $\{16\}$. ***Answer***

Example 2 Solve $\frac{1}{2}x + 3 = 9$.

Solution

$\frac{1}{2}x + 3 = 9$

$\frac{1}{2}x + 3 - 3 = 9 - 3$ — To undo the addition of 3 to $\frac{1}{2}x$, subtract 3 from each side.

$\frac{1}{2}x = 6$

$2\left(\frac{1}{2}x\right) = 2 \cdot 6$ — To undo the multiplication of x by $\frac{1}{2}$, multiply each side by 2, the reciprocal of $\frac{1}{2}$.

$x = 12$

$\therefore$ the solution set is $\{12\}$. ***Answer***

Example 3 Solve $\frac{w-5}{9} = 2$.

Solution 1

$$\frac{w-5}{9} = 2$$
$$9\left(\frac{w-5}{9}\right) = 9 \cdot 2$$
$$w - 5 = 18$$
$$w - 5 + 5 = 18 + 5$$
$$w = 23$$

$\therefore$ the solution set is $\{23\}$. ***Answer***

Solution 2 (condensed)

$$\frac{w-5}{9} = 2$$
$$w - 5 = 18$$
$$w = 23$$

$\therefore$ the solution set is $\{23\}$. ***Answer***

Examples 4 and 5 show that it is sometimes necessary to use the distributive property and simplify one or both sides of an equation as the first step in solving it.

Example 4 Solve $32 = 7a + 9a$.

Solution 1

$$32 = 7a + 9a.$$
$$32 = 16a$$
$$\frac{32}{16} = \frac{16a}{16}$$
$$2 = a$$

$\therefore$ the solution set is $\{2\}$. ***Answer***

Solution 2 (condensed)

$$32 = 7a + 9a$$
$$32 = 16a$$
$$2 = a$$

$\therefore$ the solution set is $\{2\}$. ***Answer***

Example 5 Solve $4(y + 8) - 7 = 15$.

Solution 1

$$4(y + 8) - 7 = 15$$
$$4y + 32 - 7 = 15$$

{Use the distributive property and simplify the left side.

$$4y + 25 = 15$$
$$4y + 25 - 25 = 15 - 25$$
$$4y = -10$$
$$\frac{4y}{4} = \frac{-10}{4}$$
$$y = -\frac{10}{4} = -\frac{5}{2}$$

$\therefore$ the solution set is $\left\{-\frac{5}{2}\right\}$. ***Answer***

Solution 2 (condensed)

$$4(y + 8) - 7 = 15$$
$$4y + 32 - 7 = 15$$
$$4y + 25 = 15$$
$$4y = -10$$
$$y = -\frac{10}{4} = -\frac{5}{2}$$

$\therefore$ the solution set is $\left\{-\frac{5}{2}\right\}$. ***Answer***

At the top of the next page you will find two helpful tips for solving an equation in which the variable is on one side.

1. Simplify each side of the equation as needed.
2. If the side containing the variable involves a certain order of operations, apply the inverse operations in the opposite order.

Oral Exercises

Describe how you would solve each equation.

Sample 1 $\frac{1}{5}x + 2 = -1$

Solution First subtract 2 from each side; then multiply each side by 5.

Sample 2 $14 = \frac{2m}{3}$

Solution 1 Multiply each side by 3; then divide each side by 2.

Solution 2 Multiply each side by $\frac{3}{2}$.

1. $5y + 3 = 18$ **2.** $3y - 4 = 14$ **3.** $\frac{1}{4}a - 2 = -3$

4. $-\frac{1}{2}b + 9 = 5$ **5.** $-6 + \frac{y}{5} = 5$ **6.** $4 = -4 - \frac{x}{3}$

7. $\frac{3n}{4} = -12$ **8.** $\frac{-5z}{9} = 30$ **9.** $5x + 4x = 18$

10. $-4n + 7n = -36$ **11.** $\frac{7}{8}s + 2 = 16$ **12.** $1 - \frac{4}{5}n = -19$

13. $\frac{y-2}{3} = 7$ **14.** $\frac{2}{3}w + 4 = 5$ **15.** $1 - \frac{2}{9}v = 4$

16. $5 = c + 8c - 4$ **17.** $10p - p + 2 = -7$ **18.** $-4(m + 12) = 36$

Written Exercises

Solve.

A

1. $2x - 1 = 11$ **2.** $3y - 8 = 16$ **3.** $4n + 9 = -3$

4. $5c + 7 = -28$ **5.** $-2x + 5 = 19$ **6.** $-8y - 11 = 13$

7. $\frac{1}{2}x + 7 = 6$ **8.** $\frac{2}{3}p - 7 = 17$ **9.** $\frac{2x}{3} = 8$

10. $\frac{4y}{5} = 28$ **11.** $\frac{x+5}{3} = 7$ **12.** $\frac{z-5}{4} = 8$

Solve.

13. $\frac{1-x}{2} = 7$
14. $\frac{5-n}{3} = 4$
15. $7x - 4x = 54$
16. $3y - 7y = 28$
17. $-3n - 5n = 0$
18. $2a - 11a = -27$
19. $y - 7 + 4y = 13$
20. $2x + 5 - 7x = 15$
21. $0 = n - 15 - 4n$
22. $0 = p + 18 + 5p$
23. $2(x - 4) = 22$
24. $3(y - 7) = 27$
25. $35 = 5(n + 2)$
26. $20 = 4(x + 3)$
27. $y + 5 - 4y = -10$
28. $4w - 3w + 2w = 24$
29. $15 = 8x - 5 + 2x$
30. $32 = 2n - 3n + 5n$

B

31. $-\frac{1}{2}(x + 4) = 16$
32. $\frac{3}{5}(x + 2) = 12$
33. $21 = -\frac{3}{2}(x - 2)$
34. $66 = -\frac{6}{5}(s + 3)$
35. $3(a - 5) + 19 = -2$
36. $2(b + 8) - 9 = 5$
37. $-3 = 4(k + 7) - 15$
38. $3 = 7(h - 2) + 17$
39. $4c + 3(c - 2) = -34$
40. $d + 4(d + 6) = -1$
41. $\frac{2x - 1}{3} = 5$
42. $\frac{4y + 3}{7} = 9$
43. $0 = \frac{8 - 2x}{5}$
44. $7 = \frac{4 + 9y}{7}$
45. $1 - \frac{3}{4}(v + 2) = -5$
46. $9 - \frac{4}{5}(u - 3) = 1$
47. $-9 - 3(2q - 1) = -18$
48. $-10 + 4(3p + 10) = 18$
49. $-2 = 4(s + 8) - 3s$
50. $-7 = 3(t - 5) - t$
51. $(x - 13) - (x - 5) + 2x = 0$
52. $(5 - y) + (6 - y) - (5 - y) = 0$
53. $b - (1 - 2b) + (b - 3) = -4$
54. $(c + 3) - 2c - (1 - 3c) = 2$

C

55. $5m - 3[7 - (1 - 2m)] = 0$
56. $\frac{1}{5}[4(k + 2) - (3 - k)] = 4$
57. $5(g - 7) + 2[g - 3(g - 5)] = 0$
58. $7n + 2[3(1 - n) - 2(1 + n)] = 14$
59. $3|n| - (2|n| - 2) = 9$
60. $(9|x| + 3) - 5|x| - 3 = 12$

Mixed Review Exercises

Solve.

1. $\frac{1}{7}x = -23$
2. $\frac{x}{8} = \frac{3}{4}$
3. $\frac{1}{5}x = 3\frac{1}{5}$
4. $-3 + x = 1$
5. $2x + 7 = 13$
6. $51 = y + 17$
7. $-12 + x = -20$
8. $32 - x = 36$
9. $-0.25x = 1$
10. $2.3 = x + 1$
11. $0 = 3x$
12. $18y = 360$

Computer Exercises

1. Write a BASIC program to solve an equation of the form $Ax + B = C$, where the values of A, B, and C are entered with INPUT statements. Use the program to solve the following equations.

a. $7x + 8 = 64$ **b.** $3x - 2 = -8$ **c.** $\frac{2}{5}x + 11 = 7$

d. $\frac{3}{2}x + 4 = 13$ **e.** $\frac{1}{4}x - 9 = 0$ **f.** $\frac{3}{10}x + 10.7 = 14$

2. Use the program from Exercise 1 to solve $0x + 9 = 12$. What happens? What is the correct solution? Modify the program from Exercise 1 to print an appropriate response if the value of A is 0.

3. Modify the program from Exercise 1 to solve an equation of the form $A|x| + B = C$. Use this program to solve the following equations.

a. $|x| + 8 = 10$ **b.** $|x| + 10 = 8$ **c.** $6|x| + 2 = 5$

Self-Test 1

Vocabulary equivalent equations (p. 96)
transformation by substitution (p. 96)
transformation by addition (p. 96)
transformation by subtraction (p. 96)
transformation by multiplication (p. 102)
transformation by division (p. 102)
inverse operations (p. 107)

Solve.

1. $n - 32 = 6$ **2.** $19 + y = 61$ **Obj. 3-1, p. 95**

3. $625 = 5y$ **4.** $-\frac{1}{2}x = 5\frac{1}{2}$ **Obj. 3-2, p. 102**

5. A rectangle is three times as long as it is wide. If its perimeter is 48 cm, find its dimensions. Draw a diagram first.

6. $3x - 4 = 17$ **7.** $\frac{1}{2}y + 4 = 16$ **Obj. 3-3, p. 107**

Check your answers with those at the back of the book.

Calculator Key-In

You can use a calculator to check whether 16 is a solution of $5n - 9 = 71$. If it is, the calculator will display 71 when you enter $5 \times 16 - 9$.

Exercises

Is the given number a solution of the equation?

1. $6x - 7 = 21$; 3.5 **2.** $22x + 5 = 60$; 2.5 **3.** $3.9x - 11.2 = 4.6$; 4.2

3-4 Using Equations to Solve Problems

Objective To use the five-step plan to solve word problems.

The skills that you have gained in solving equations can often help you to solve word problems. Use the five-step plan on page 27 as a guide.

Example 1 Lynne took a taxicab from her office to the airport. She had to pay a flat fee of \$2.05 plus \$.90 per mile. The total cost was \$5.65. How many miles was the taxi trip?

Solution

Step 1 The problem asks for the number of miles traveled in the taxi.

Step 2 Let m = the number of miles.
Then $90m$ = the mileage cost in cents.

Step 3 Flat fee + mileage cost = total cost

$$205 + 90m = 565$$

Step 4 Solve.

$$90m = 360$$
$$m = 4$$

Step 5 *Check:* 4 miles at \$.90 per mile: 4(\$.90) = \$3.60

Flat fee + mileage cost: \$2.05 + \$3.60 = \$5.65 ✓

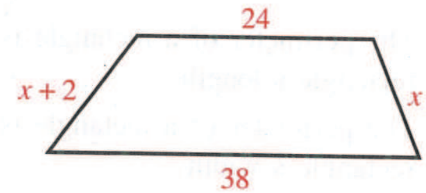
∴ the taxi trip was 4 mi. ***Answer***

Example 2 shows that some word problems can be solved more easily if you first draw a diagram.

Example 2 The perimeter of a trapezoid is 90 cm.
The parallel bases are 24 cm and 38 cm long.
The lengths of the other two sides are consecutive odd integers.
What are the lengths of these other two sides?

Solution

Step 1 Draw a diagram to help you understand the problem.



Step 2 Use x and $x + 2$ to represent the unknown lengths of the sides.

Step 3
$$\text{perimeter} = 90$$
$$38 + x + 24 + (x + 2) = 90$$

Step 4
$$2x + 64 = 90$$
$$2x = 26$$
$$x = 13 \text{ and } x + 2 = 15$$

Step 5 *Check:* Is the sum of the lengths of the sides 90 cm?
$38 + 13 + 24 + 15 = 90$ ✓

$\therefore$ the required lengths are 13 cm and 15 cm. ***Answer***

Problems

Solve each problem using the five-step plan to help you.

A

1. The sum of 38 and twice a number is 124. Find the number.
2. Five more than three times a number is 197. Find the number.
3. Four less than half of a number is 17. Find the number.
4. When one third of a number is decreased by 11, the result is 38. Find the number.
5. Four more than two thirds of a number is 22. Find the number.
6. Eight less than three quarters of a number is 91. Find the number.
7. Find three consecutive integers whose sum is 171.
8. Find three consecutive odd integers whose sum is 105.
9. Find four consecutive even integers whose sum is 244.
10. Find five consecutive integers whose sum is 195.
11. Burt's Burger Barn sold 495 hamburgers today. The number sold with cheese was half the number sold without cheese. How many of each kind were sold?
12. A company added a new oil tank that holds 350 barrels of oil more than its old oil tank. Together they hold 3650 barrels of oil. How much does each tank hold?
13. Brian has \$88 in his savings account. If he saves \$3.50 per week, how long will it take him to have \$200 in his account?
14. A 1000 L tank now contains 240 L of water. How long will it take to fill the tank using a pump that pumps 25 L per minute?

Solve each problem using the five-step plan. In Exercises 15–26, draw a diagram to help you.

15. The perimeter of a rectangle is 332 cm and the width is 76 cm. Find the rectangle's length.

16. The perimeter of a rectangle is 408 cm and the length is 134 cm. Find the rectangle's width.

17. In an isosceles triangle, there are two sides, called *legs,* with the same length. The third side is called the *base*. If an isosceles triangle has perimeter 345 cm and base length 85 cm, what is the length of each leg?

18. The length of a rectangle is 7 cm more than the width. The perimeter is 78 cm. Find the rectangle's dimensions.

19. The width of a rectangle is 15 cm less than the length. The perimeter is 98 cm. Find the rectangle's dimensions.

20. The longest side of a triangle is twice as long as the shortest side and the remaining side is 25 cm. If the perimeter is 70 cm, find the lengths of the sides of the triangle.

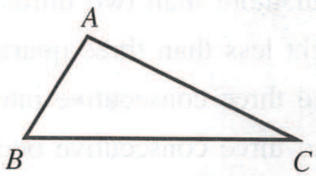
21. A rectangle's length is 8 cm more than three times its width. If the perimeter is 128 cm, find the length and the width.

22. A triangle has sides with lengths in centimeters that are consecutive even integers. Find the lengths if the perimeter is 186 cm.

B **23.** In any triangle, the sum of the measures of the angles is 180°. In $\triangle ABC$, $\angle A$ is three times as large as $\angle B$ and also 16° larger than $\angle C$. Find the measure of each angle.

24. In any triangle, the sum of the measures of the angles is 180°. In $\triangle ABC$, $\angle A$ is twice as large as $\angle B$. $\angle B$ is 4° larger than $\angle C$. Find the measure of each angle.

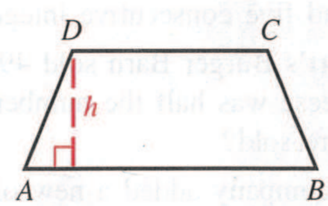
25. In $\triangle ABC$, $\overline{AB}$ is 9 cm shorter than $\overline{AC}$, while $\overline{BC}$ is 3 cm longer than $\overline{AC}$. If the perimeter of the triangle is 48 cm, find the lengths of the three sides.



26. In isosceles trapezoid $ABCD$, the longer base, $\overline{AB}$, is one and one half times as long as the shorter base, $\overline{CD}$. The other two sides, $\overline{AD}$ and $\overline{BC}$, are both 13 cm long.

a. If the perimeter is 76 cm, find the lengths of $\overline{AB}$ and $\overline{CD}$.

b. If the height of the trapezoid is 12 cm, find its area.
(*Hint:* Area $= \frac{1}{2} \times$ height $\times$ sum of base lengths)



27. Theo has \$5 more than Denise and Denise has \$11 more than Rudy. Together they have \$45. How much money does each have?

28. Chandra has twice as much money as Nora. Nora has \$6 less than Lian. Together they have \$54. How much money does each have?

29. In one day, Machine A caps twice as many bottles as Machine B. Machine C caps 500 more bottles than Machine A. The three machines cap a total of 40,000 bottles in a day. How many bottles does each of the machines cap in one day?

30. The total cost of a sandwich, a glass of milk, and an apple is $3.50. The milk costs one and a half times as much as the apple. The sandwich costs $1.40 more than the apple. What is the price of each?

C **31.** With the major options package and destination charge, a sports car cost $24,416. The base price of the car was ten times the price of the major options package and fifty times the destination charge. What was the base price of the car?

32. On the first of three tests, Keiko scored 72 points. On the third test, her score was 1 point more than on the second. Her average on the three tests was 83. What were her scores on the second and third tests?

33. The absolute value of the sum of -7 and twice a number is 23. Find the number. (*Hint:* There are two answers.)

Mixed Review Exercises

Solve.

1. $-5 + y = 1$	**2.** $x - 1.5 = -3$	**3.** $y + 8 = 21$	**4.** $\frac{2}{5}y = 2$
5. $-21 = \frac{c}{3}$	**6.** $-\frac{1}{5}x = 20$	**7.** $41 = y + 11$	**8.** $x - 20 = 37$
9. $0 = 1.75y$	**10.** $5y + 6 = 16$	**11.** $2x - 5 = 15$	**12.** $2(a - 3) + 6 = -3$

Historical Note / *Variables*

Until the sixteenth century, unknown quantities were represented by words such as "heap," "root," or "thing." Eventually, abbreviations for such words, as well as drawings of squares and cubes, were used to symbolize unknowns.

In the late sixteenth century, a French lawyer, Francois Vietá, who enjoyed studying algebra during his leisure hours, began using vowels for unknowns. An English mathematician, Thomas Harriot, later adopted lowercase letters to stand for variables. In 1637, René Descartes, a French mathematician and philosopher, began using the final letters of the alphabet to represent unknowns.

3-5 Equations with the Variable on Both Sides

Objective To solve equations with the variable on both sides.

In the first four lessons of this chapter, the variable appeared on just one side of a given equation. In this lesson, the variable may occur on both sides of the equation. Since variables represent numbers, you may transform an equation by adding a variable expression to each side or by subtracting a variable expression from each side. Then solve the resulting equation as you have in earlier lessons.

Example 1 Solve $6x = 4x + 18$.

Solution 1

$$6x = 4x + 18$$
$$6x - 4x = 4x + 18 - 4x \qquad \text{Subtract } 4x \text{ from each side.}$$
$$2x = 18$$
$$\frac{2x}{2} = \frac{18}{2}$$
$$x = 9$$

Solution 2 (condensed)

$6x = 4x + 18$ *Check:* $6 \cdot 9 \stackrel{?}{=} 4 \cdot 9 + 18$

$2x = 18$ $54 \stackrel{?}{=} 36 + 18$

$x = 9$ $54 = 54$ ✓

∴ the solution set is {9}. ***Answer***

Example 2 Solve $3y = 15 - 2y$.

Solution

$3y = 15 - 2y$ Add $2y$ to both sides.

$5y = 15$

$y = 3$ ∴ the solution set is {3}. ***Answer***

Example 3 Solve: **a.** $\frac{4}{5}x + 3 = x$ **b.** $\frac{8 + x}{9} = x$

Solution

a.

$$3 = x - \frac{4}{5}x$$
$$3 = x\left(1 - \frac{4}{5}\right)$$
$$3 = \frac{1}{5}x$$
$$15 = x$$

∴ the solution set is {15}. ***Answer***

b.

$$8 + x = 9x$$
$$8 = 8x$$
$$1 = x$$

∴ the solution set is {1}. ***Answer***

Example 4 Solve $7(a-2)-6=2a+8+a$.

Solution

$$7(a-2)-6=2a+8+a$$
$$7a-14-6=2a+8+a$$

First use the distributive property and simplify both sides.

$$7a-20=3a+8$$
$$4a-20=8$$
$$4a=28$$
$$a=7$$

$\therefore$ the solution set is $\{7\}$. ***Answer***

It is possible that an equation may have *no* solution, or that it may be satisfied by *every* real number. Examples 5 and 6 illustrate these cases.

Example 5 Solve $3(1-r)+5r=2(r+1)$.

Solution

$$3-3r+5r=2r+2$$
$$3+2r=2r+2$$
$$3+2r-2r=2r+2-2r$$
$$3=2$$

The given equation is equivalent to the false statement $3=2$.
$\therefore$ the equation has no solution. ***Answer***

We call the set with no members the **empty set,** or the **null set.** It is denoted by the symbol $\emptyset$. The solution set of the equation in Example 5 is $\emptyset$.

Example 6 Solve $\frac{1}{3}(12x-21)=4x-7$.

Solution

$$4x-7=4x-7$$

The given equation is equivalent to $4x-7=4x-7$, which is satisfied by every real number. $\therefore$ the solution set is {real numbers}. ***Answer***

An equation that is true for every value of the variable is called an **identity.** The equation in Example 6 is an identity.

Oral Exercises

Solve. If the equation is an identity or if it has no solution, say so.

1. $4x=3x+5$
2. $4n+10=5n$
3. $8r+1=9r$
4. $2p-1=3p$
5. $7+b=b+7$
6. $2b=6+2b$
7. $4a=2a+a$
8. $8k=k$
9. $3s=s-2$
10. $3n+4=3n+5$
11. $5(x-2)=5x-10$
12. $3(w+1)=2w$

Written Exercises

Solve each equation. If the equation is an identity or if it has no solution, write *identity* or *no solution*.

A

1. $5n = 2n + 6$
2. $8a = 2a + 30$
3. $y = 24 - 3y$
4. $2b = 80 - 8b$
5. $12n = 34 - 5n$
6. $3x = 27 - 15x$
7. $30 = 8 - 2x$
8. $51 = 9 - 3x$
9. $51a - 56 = 44a$
10. $39c + 78 = 33c$
11. $98 - 4b = -11b$
12. $-7a = -12a - 65$
13. $4n + 5 = 6n + 7$
14. $5p - 9 = 2p + 12$
15. $3p - 8 = 13 - 4p$
16. $89 + x = 2 - 2x$
17. $71 - 5x = 9x - 13$
18. $5n + 1 = 5n - 1$
19. $2(x - 6) = 3x$
20. $4(y - 6) = 7y$
21. $8(5 - n) = 2n$
22. $7(2 - m) = 3m$
23. $\frac{1}{2}x + 5 = x$
24. $\frac{2}{3}x - 7 = x$
25. $\frac{4 + y}{5} = y$
26. $\frac{x - 2}{3} = x$
27. $\frac{9 - 2y}{7} = y$
28. $\frac{6 - 4y}{2} = y$
29. $\frac{4n - 28}{3} = 2n$
30. $\frac{23 - 11c}{7} = 5c$

B

31. $\frac{1}{3}(12 - 6x) = 4 - 2x$
32. $\frac{1}{4}(20 - 4a) = 6 - a$
33. $5(2 + n) = 3(n + 6)$
34. $3(30 + s) = 4(s + 19)$
35. $5u + 5(1 - u) = u + 8$
36. $2(g - 2) - 4 = 2(g - 3)$
37. $3(m + 5) - 6 = 3(m + 3)$
38. $3(2 + v) - 4v = v + 16$
39. $3(5y + 2) - y = 2(y - 3)$
40. $4(3y - 1) + 13 = 5y + 2$
41. $6r - 2(2 - r) = 4(2r - 1)$
42. $5x + 2(1 - x) = 2(2x - 1)$
43. $3 + 4(p + 2) = 2p + 3(p + 4)$
44. $4(a + 2) = 14 - 2(3 - 2a)$

C

45. $3x + 2[1 - 3(x + 2)] = 2x$
46. $2[5(w + 3) - (w + 1)] = 3(1 + w)$
47. $5(2m + 3) - (1 - 2m) = 2[3(3 + 2m) - (3 - m)]$
48. $3(r + 1) - [2(3 - 2r) - 3(3 - r)] = 2(r + 5) - 4$

Problems

Solve.

A

1. Find a number that is 96 greater than its opposite.
2. Find a number that is 38 less than its opposite.
3. Find a number whose product with 9 is the same as its sum with 56.
4. Find a number that is 68 greater than three times its opposite.

5. Three times a number, decreased by 8, is the same as twice the number, increased by 15. Find the number.

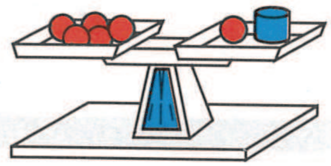
6. Four times a number, increased by 25, is 13 less than six times the number. Find the number.

7. The greater of two consecutive integers is 15 more than twice the smaller. Find the integers.

8. The greater of two consecutive even integers is 20 more than twice the smaller. Find the integers.

9. Lyle shot three times as many baskets as Cliff, while Kyle shot 12 more baskets than Cliff. If Lyle and Kyle shot the same number of baskets, how many baskets did each of them shoot?

10. Dionne has six steel balls of equal mass. If she puts five of them in one pan of a beam balance and one ball and a 100 g mass in the other pan, the pans balance each other. What is the mass of each steel ball?



B 11. The sum of two numbers is 15. Three times one of the numbers is 11 less than five times the other. Find the numbers.

12. The difference of two integers is 9. Five times the smaller is 7 more than three times the larger. Find the numbers.

13. The lengths of the sides of a triangle are consecutive even integers. Find the length of the longest side if it is 22 units shorter than the perimeter.

14. The length of a rectangle is twice the width. The perimeter is 84 cm more than the width. Make a diagram and find the rectangle's dimensions.

15. Mei's salary starts at $16,000 per year with annual raises of $1500. Janet's starting salary is $19,300 with annual raises of $950. After how many years will the two women be earning the same salary?

16. A 2000 L tank containing 550 L of water is being filled with water at the rate of 75 L per minute from a full 1600 L tank. How long will it be before the two tanks have the same amount of water?

17. Eric has twice as much money as Marcia, who has $175 less than Laurel. But Laurel has as much money as Eric and Marcia have together. How much money does each person have?

18. A boat weighs 1500 lb more than its motor and 1900 lb more than its trailer. Together the boat and motor weigh five times as much as the trailer. How much does the boat weigh?

19. Show that it is impossible for three consecutive integers to have a sum that is 200 more than the smallest integer.

20. Is it possible for four consecutive even numbers to have a sum that is ten more than the sum of the smallest two numbers? If so, tell how many solution(s) there are. If there are no solutions, tell why not.

Mixed Review Exercises

Simplify.

1. $4 + \left(-\frac{1}{2}\right) + \left(-\frac{3}{2}\right)$ **2.** $-2\frac{3}{5} + 1\frac{1}{5}$ **3.** $-212 - (-13)$

4. $17x + (-3)x - 5$ **5.** $-5y + 6 + 20y + 12$ **6.** $7(-3)(-10)(-2)$

Solve.

7. $-3 - x = 9$ **8.** $5 - (1 + z) = 3$ **9.** $4x = -392$

10. $\frac{1}{2}x = 4\frac{1}{2}$ **11.** $\frac{x}{7} = 6$ **12.** $-12\frac{2}{3} = -\frac{1}{3}x$

Computer Exercises

For students with some programming experience.

Write a BASIC program to solve an equation of the form Ax + B = Cx + D, where the values of A, B, C, and D are entered with INPUT statements. Be sure that the program prints an appropriate message for identities and for equations having no solution. Use the program to solve the following equations.

1. $3x + 4 = 5x + 10$ **2.** $\frac{1}{2}x + 1 = -2x + 11$

3. $4x - 7 = 3 + 4x$ **4.** $\frac{1}{2}x + 5 = 5 + \frac{1}{2}x$

5. $3x - 13 = \frac{2}{5}x$ **6.** $3 - x = 4x - 3$

Self-Test 2

Vocabulary empty set (p. 117) identity (p. 117)
null set (p. 117)

Solve.

1. A \$48 sweater costs \$6 more than twice as much as the shirt that goes with it. How much does the shirt cost? **Obj. 3-4, p. 112**

2. $25 - 4n = n$ **3.** $3(x + 1) = 2(x + 5)$ **Obj. 3-5, p. 116**

4. Hilary has three times as much money as Paul. Jeff has \$4 less than Hilary and \$5 more than Paul. How much money does each have?

Check your answers with those at the back of the book.

Extending Your Problem Solving Skills

3-6 Problem Solving: Using Charts

Objective To organize the facts of a problem in a chart.

Using a chart to organize the facts of a problem can be a helpful problem solving strategy.

Example 1 Organize the given information in a chart:
A roll of carpet 9 ft wide is 20 ft longer than a roll of carpet 12 ft wide.

Solution 1

	Width	Length
First roll	9	l
Second roll	12	$l - 20$

Solution 2

	Width	Length
First roll	9	$l + 20$
Second roll	12	l

Example 2 Solve the problem using the two given facts.
Find the number of Calories in an apple and in a pear.
(1) A pear contains 30 Calories more than an apple.
(2) Ten apples have as many Calories as 7 pears.

Solution

Step 1 The problem asks for the number of Calories in an apple and in a pear.

Step 2 Let $a =$ the number of Calories in an apple.
Then $a + 30 =$ the number of Calories in a pear.

	Calories per fruit ×	Number of fruit =	Total Calories
Apple	a	10	$10a$
Pear	$a + 30$	7	$7(a + 30)$

Step 3 Calories in 10 apples = Calories in 7 pears

$$10a = 7(a + 30)$$

Step 4

$$10a = 7a + 210$$
$$3a = 210$$
$$a = 70 \text{ and } a + 30 = 100$$

(Solution continues on the next page.)

Step 5 *Check:* (1) 100 Calories is 30 more than 70 Calories. ✓
(2) Ten apples have $10 \cdot 70$, or 700 Calories and seven pears have $7 \cdot 100$, or 700 Calories. ✓

$\therefore$ there are 70 Calories in an apple and 100 Calories in a pear. ***Answer***

Oral Exercises

Organize the given information by completing each chart.

1. A swimming pool 25 m long is 13 m narrower than a pool 50 m long.

a.

	Length	Width
1st pool	25	?
2nd pool	50	w

b.

	Length	Width
1st pool	25	w
2nd pool	50	?

2. In game 1, Ellen scored twice as many points as Jody. In game 2 Ellen scored ten fewer points than she did in game 1, while Jody scored 12 more points than she did in game 1.

	Game 1 points	Game 2 points
Ellen	?	?
Jody	m	?

3. Use the two given facts to complete the chart. What equation would you write to find the amount of protein in a scrambled egg?
(1) An egg scrambled with butter and milk has one more gram of protein than an egg fried in butter.
(2) Ten scrambled eggs have as much protein as a dozen fried eggs.

	Protein per egg	× Number of eggs	= Total protein
Scrambled egg	?	10	?
Fried egg	x	?	?

Problems

Solve each problem using the two given facts. If a chart is given, first copy and complete the chart to help you solve the problem.

A **1.** Find the number of full 8 hour shifts that Maria worked last month.
(1) She worked twice as many 6 hour shifts as 8 hour shifts.
(2) She worked a total of 280 hours.

(Chart on next page)

	Hours per shift × Number of shifts = Total hours worked		
6 h shift	?	?	?
8 h shift	?	x	?

2. Find the number of round-trip commuter rail tickets sold.
(1) Thirty times as many round-trip tickets as 12-ride tickets were sold.
(2) The total number of tickets sold represented 1440 rides.

	Rides per ticket × Number of tickets sold = Total rides		
12-ride ticket	?	n	?
Round-trip ticket	?	?	?

3. Find the total weight of the boxes of pecans in a shipment of 3 lb boxes of pecans and 2 lb boxes of walnuts.
(1) There were 24 fewer 2 lb boxes of walnuts than 3 lb boxes of pecans.
(2) The total weight of the shipment was 462 lb.

	Weight per box × Number of boxes = Total Weight		
Pecans	?	?	?
Walnuts	?	?	?

4. Find the amount of time Joel spent watching space adventure movies.
(1) He saw twice as many $1\frac{1}{2}$ h space movies as he did 2 h mysteries.
(2) He spent a total of 15 h watching movies.

	Movie length × Number of movies = Total time		
Space movies	?	?	?
Mystery movies	?	?	?

5. Find the number of Calories in an orange and in a peach.
(1) An orange has 30 Calories more than a peach.
(2) Thirteen peaches have as many Calories as 7 oranges.

6. Find the number of Calories in a stalk of celery and in a carrot.
(1) A carrot has 13 Calories more than a celery stalk.
(2) Five carrots and ten celery stalks have only 170 Calories.

Solve. Use a chart to help you solve the problem.

7. The length of a red rectangle is 15 cm more than its width w. A blue rectangle, which is 5 cm wider and 2 cm shorter than the red one, has perimeter 72 cm. Make a sketch of the rectangles expressing all dimensions in terms of w. Then find the dimensions of each rectangle.

Solve. Use a chart to help you solve the problem.

8. The length of a rectangle is twice its width w. A second rectangle, which is 8 cm longer and 3 cm narrower than the first rectangle, has perimeter 154 cm. Make a sketch of the rectangles expressing all dimensions in terms of w. Then find the dimensions of each rectangle.

B **9.** Brian O'Reilly earns twice as much each week as a tutor than he does pumping gas. His total weekly wages are $150 more than that of his younger sister. She earns one quarter as much as Brian does as a tutor. How much does Brian earn as a tutor?

10. Mona Yahuso earns three times as much as an actuary as she does as a writer. Her total income is $40,000 more than that of her brother. He earns half as much as Mona does as an actuary. What is Mona's salary as an actuary?

11. A roll of carpet 9 ft wide is 30 ft longer than a roll of carpet 15 ft wide. Both rolls have the same area. Make a sketch of the unrolled carpets and find the dimensions of each.

12. Leo's garden, which is 6 m wide, has the same area as Jen's garden, which is 8 m wide. Find the lengths of the two rectangular gardens if Leo's garden is 3 m longer than Jen's garden. First make a sketch.

13. In March, Rodney sold twice as many cars as Greg. In April, Rodney sold 5 fewer cars than he did in March, while Greg sold 3 more cars than he did in March. If they sold the same number of cars in April, how many cars did each sell in March?

14. In one basketball game Maria scored three times as many points as Holly. In the next game, Maria scored 7 fewer points than she did in the first game, while Holly scored 9 more points than she did in the first game. If they scored the same number of points in the second game, how many points did each score in the first game?

15. Paula mixed 2 cups of sunflower seeds and 3 cups of raisins to make a snack for a hike. She figured that the mixture would provide her with 2900 Calories of food energy. Find the number of Calories per cup of raisins if it is 400 less than the number of Calories per cup of sunflower seeds.

16. The Eiffel Tower is 497 ft taller than the Washington Monument. If each of the monuments were 58 ft shorter, the Eiffel Tower would be twice as tall as the Washington Monument. How tall is each?

17. The upper Angel Falls, the highest waterfall on Earth, are 750 m higher than Niagara Falls. If each of the falls were 7 m lower, the upper Angel Falls would be 16 times as high as Niagara Falls. How high is each waterfall?

18. Nine cartons of juice cost the same as 5 fruit cups. Also, one fruit cup costs 50¢ more than one bowl of soup, while one bowl of soup costs 50¢ more than one carton of juice. What would be the cost of each item: a carton of juice, a fruit cup, and a bowl of soup?

19. One serving ($\frac{1}{2}$ cup) of cooked peas contains 45 more Calories than one serving of cooked carrots and 50 more Calories than one serving of cooked green beans. If one serving of carrots and three servings of green beans contain the same number of Calories as one serving of peas, how many Calories are there in one serving of peas?

C **20.** The cross-country teams of East High and West High run against each other twice each fall. At the first meet, East's score was two thirds of West's score. At the second meet, East's score increased by seven points and West's score decreased by seven points. In the second meet West's score was three less than East's score. How many points did each team score in each meet?

Mixed Review Exercises

Solve.

1. $25z = 600$

2. $6 = \frac{2}{5}x$

3. $12z - 3z = 0$

4. $195 = 3x$

5. $7y + 3 = 24$

6. $-12 + 3y = -36$

7. $5x - 2x = 21$

8. $4(x + 2) = 6x$

9. $7x - 11 = 2x + 44$

10. $41 - x = -1 - 7x$

11. $-x = 2x - 54$

12. $5(y + 2) - 3(y - 1) = -27$

Career Note / *Astronomer*

Modern astronomers make few direct observations with telescopes. Instead, they use mathematics and physics to explore the nature of the universe. Their theories are described by mathematical equations that are tested on computers using data from observatories.

Observatories often gather data with radio telescopes and spectroscopes. Radio telescopes are used to detect the invisible x rays and radio waves that are emitted by stars. Spectroscopes, on the other hand, are used to separate a star's visible light into its various wave lengths, to form the star's spectral pattern.

With the aid of a computer, as shown in the photo, an astronomer can analyze a star's spectral pattern to determine whether the star is moving toward or away from the Earth.

Most astronomers work in universities or government space centers as teachers or researchers. They are highly trained in mathematics and physics and most have a Ph.D. in astronomy.

3-7 Cost, Income, and Value Problems

Objective To solve problems involving cost, income, and value.

The word problems in this lesson involve cost, income, and value. Organizing the given facts in a chart will help you to solve such problems. The following formulas will be useful in setting up your charts.

Cost = number of items × price per item

Income = hours worked × wage per hour

Total value = number of items × value per item

Example Tickets for the senior class play cost \$6 for adults and \$3 for students. A total of 846 tickets worth \$3846 were sold. How many student tickets were sold?

Solution

Step 1 The problem asks for the number of student tickets sold.

Step 2 Let x = the number of student tickets sold.
Then $846 - x$ = the number of adult tickets sold.

Make a chart.

	Number ×	Price per ticket =	Cost
Student	x	3	$3x$
Adult	$846 - x$	6	$6(846 - x)$

Step 3 The only fact not recorded in the chart is that the total cost of the tickets was \$3846. Write an equation using this fact.

Student ticket cost + adult ticket cost = 3846

$$3x + 6(846 - x) = 3846$$

Step 4

$$3x + 5076 - 6x = 3846$$
$$5076 - 3x = 3846$$
$$-3x = -1230$$
$$x = 410 \leftarrow \text{student tickets}$$
$$846 - x = 436 \leftarrow \text{adult tickets}$$

Step 5 *Check:* 410 student tickets at \$3 each cost \$1230.
436 adult tickets at \$6 each cost \$2616.

The total number of tickets is 410 + 436, or 846.
The total cost of the tickets is \$1230 + \$2616, or \$3846. ✓

∴ 410 student tickets were sold. ***Answer***

Oral Exercises

Read each problem and complete the chart. Then give an equation that can be used to solve the problem.

1. Marlee makes \$5 an hour working after school and \$6 an hour working on Saturdays. Last week she made \$64.50 by working a total of 12 hours. How many hours did she work on Saturday?

	Hours worked ×	Wage per hour =	Income
Saturdays	s	?	?
Weekdays	?	?	?

2. Ernesto purchased 100 postage stamps worth \$9.90. Half of them were 1¢ stamps, and the rest were 14¢ and 22¢ stamps. How many 22¢ stamps did he buy? (*Hint:* In your equation, use 990¢ instead of \$9.90.)

	Number ×	Price =	Cost
1¢ stamps	?	?	?
14¢ stamps	?	?	?
22¢ stamps	x	?	?

Problems

Solve. Copy and complete the chart first.

A

1. Thirty students bought pennants for the football game. Plain pennants cost \$4 each and fancy ones cost \$8 each. If the total bill was \$168, how many students bought the fancy pennants?

	Number ×	Price =	Cost
Fancy	f	?	?
Plain	?	?	?

2. Adult tickets for the game cost \$4 each and student tickets cost \$2 each. A total of 920 tickets worth \$2446 were sold. How many student tickets were sold?

	Number ×	Price =	Cost
Adult	?	?	?
Student	s	?	?

3. A collection of 40 dimes and nickels is worth \$2.90. How many nickels are there? (*Hint:* In your equation, use 290¢ instead of \$2.90.)

	Number ×	Value of coin =	Total value
Dimes	?	?	?
Nickels	?	?	?

Solve. If a chart is given, copy and complete the chart first.

4. A collection of 52 dimes and nickels is worth \$4.50. How many nickels are there?

	Number ×	Value of coin (¢) =	Total value (¢)
Dimes	?	?	?
Nickels	?	?	?

5. Hans paid \$1.50 each for programs to the game. He sold all but 20 of them for \$3 each and made a profit of \$15. How many programs did he buy? (*Hint:* Profit = selling price − purchase price)

	Number ×	Price (\$) =	Cost (\$)
Bought	?	?	?
Sold	?	?	?

6. Celia bought 12 apples, ate two of them, and sold the rest at 20¢ more per apple than she paid. Her total profit was \$1.00. How much did she sell each apple for?

	Number ×	Price (¢) =	Cost (¢)
Bought	?	?	?
Sold	?	?	?

B 7. I have twice as many nickels as quarters. If the coins are worth \$4.90, how many quarters are there?

8. I have eight more quarters than dimes. If the coins are worth \$6.20, how many dimes are there?

9. The Audio Outlet purchased 60 cassette recorders, gave away three in a contest, and sold the rest at twice their purchase price. If the store's total profit was \$1188, how much did the store sell each recorder for?

10. The Alan Company bought 80 tickets for a jazz concert. After giving away 20 tickets to customers, the company sold the rest to employees at half of the purchase price. If the company absorbed a \$1000 loss on all the tickets, how much did an employee pay for a ticket?

11. A plumber makes \$4.50 per hour more than his apprentice. During an 8-hour day, their combined earnings total \$372. How much does each make per hour? (*Hint:* If you decide to use cents instead of dollars, then use 450 cents per hour and 37200 cents total earnings.)

12. Raul works 2 h daily after school, Monday through Friday. On Saturdays he works 8 h at \$2 more per hour than on weekdays. If he makes \$142 per week, how much does he make per hour on weekdays?

13. Warren has 40 coins (all nickels, dimes, and quarters) worth $4.05. He has 7 more nickels than dimes. How many quarters does Warren have?

14. Jo has 37 coins (all nickels, dimes, and quarters) worth $5.50. She has 4 more quarters than nickels. How many dimes does Jo have?

15. Rory claims: "I have $20 in quarters, half dollars, and one-dollar bills. I have twice as many quarters as half dollars, and half as many one-dollar bills as half dollars." Give a convincing argument to explain why he must be wrong.

16. Eleanor claims: "It is possible for 46 pennies and nickels to have a total value of a dollar." Is she right or wrong? Give a convincing argument to justify your answer.

C 17. Nadine has seven more nickels than Delano has dimes. If Delano gives Nadine four of his dimes, then Delano will have the same amount of money as Nadine. How much money do they have together? (Assume that Nadine has only nickels and Delano has only dimes.)

18. Natalie has some nickels, Dirk has some dimes, and Quincy has some quarters. Dirk has five more dimes than Quincy has quarters. If Natalie gives Dirk a nickel, Dirk gives Quincy a dime, and Quincy gives Natalie a quarter, they will all have the same amount of money. How many coins did each have originally?

Mixed Review Exercises

Simplify.

1. $\frac{40 \div 5 + 3}{13 - 2}$
2. $36 \div \frac{1}{6}$
3. $\frac{1}{3}(39y - 6) + 3$
4. $(-7)(5)(-1)$
5. $4(2x + 7) + 5(-x)$
6. $7(x + y) + 8(2y + x)$

Evaluate if $a = 4$, $b = 5$, and $x = 8$.

7. $\frac{5a + b}{x - 3}$
8. $\frac{ab}{4x}$
9. $3(a + b) - x \div 2$

Computer Exercises

For students with some programming experience.

Jane has a total of 12 coins, some of which are nickels and the rest dimes.

1. Write a BASIC program that uses a FOR . . . NEXT loop to print a chart showing every possible combination of dimes and nickels. The chart should also show the total value of the coins for each combination.
2. Modify the program from Exercise 1 to print the chart for K coins. The value of K should be entered with an input statement.

3-8 Proof in Algebra

Objective To prove statements in algebra.

Some of the properties stated earlier in this book are statements we assume to be true. Others are theorems. A **theorem** is a statement that is shown to be true using a logically developed argument. Logical reasoning that uses given facts, definitions, properties, and other already proved theorems to show that a theorem is true is called a **proof.** Example 1 shows how a theorem is proved in algebra.

Example 1 *Prove:* For all numbers a and b, $(a + b) - b = a$.

Proof

Statements	Reasons
1. $(a + b) - b = (a + b) + (-b)$	1. Definition of subtraction
2. $(a + b) + (-b) = a + [b + (-b)]$	2. Associative property of addition
3. $b + (-b) = 0$	3. Property of opposites
4. $a + [b + (-b)] = a + 0$	4. Substitution principle
5. $a + 0 = a$	5. Identity property of addition
6. $(a + b) - b = a$	6. Transitive property of equality

Generally, a shortened form of proof is given, in which only the *key reasons* are stated. (The substitution principle and properties of equality are usually not stated.) The proof shown in Example 1 could be shortened to the steps shown below.

Statements	Reasons
1. $(a + b) - b = (a + b) + (-b)$	1. Definition of subtraction
2. $\quad\quad = a + [b + (-b)]$	2. Associative property of addition
3. $\quad\quad = a + 0$	3. Property of opposites
4. $\quad\quad = a$	4. Identity property of addition

Example 2 *Prove:* For all real numbers a and b such that $a \neq 0$ and $b \neq 0$,

$$\frac{1}{ab} = \frac{1}{a} \cdot \frac{1}{b}. \qquad \textit{(Property of the reciprocal of a product)}$$

Proof Since $\frac{1}{ab}$ is the unique reciprocal of ab, you can prove that $\frac{1}{ab} = \frac{1}{a} \cdot \frac{1}{b}$ by showing that the product of ab and $\frac{1}{a} \cdot \frac{1}{b}$ is 1:

Statements	Reasons
1. $(ab)\left(\frac{1}{a} \cdot \frac{1}{b}\right) = \left(a \cdot \frac{1}{a}\right)\left(b \cdot \frac{1}{b}\right)$	1. Commutative and associative properties of multiplication
2. $\quad\quad = 1 \cdot 1$	2. Property of reciprocals
3. $\quad\quad = 1$	3. Identity property of multiplication

Once a theorem has been proved, it can be used as a reason in other proofs. You may refer to the Chapter Summary on page 88 for listings of properties and theorems that you can use as reasons in your proofs in the following exercises.

Oral Exercises

State the missing reasons. Assume that each variable represents any real number.

1. *Prove:* If $a = b$, then $a + c = b + c$.
(Addition property of equality)

Proof:

1. $a + c = a + c$	1. _?_ property of equality
2. $a = b$	2. Given
3. $a + c = b + c$	3. _?_ principle

2. *Prove:* If $a = b$, then $a - c = b - c$.
(Subtraction property of equality)

Proof:

1. $a = b$	1. _?_
2. Since c is a real number, $-c$ is a real number.	2. Property of opposites
3. $a + (-c) = b + (-c)$	3. _?_ property of equality (proved in Oral Exercise 1)
4. $a - c = b - c$	4. Definition of _?_

3. *Prove:* If $a = b$, then $-a = -b$.

Proof:

1. $a = b$	1. _?_
2. $a + (-b) = b + (-b)$	2. _?_ property of equality
3. $a + (-b) = 0$	3. Property of _?_
4. $-a + [a + (-b)] = -a + 0$	4. _?_ property of equality
5. $(-a + a) + (-b) = -a + 0$	5. _?_ property of addition
6. $0 + (-b) = -a + 0$	6. Property of _?_
7. $-b = -a$	7. _?_ property of addition
8. $-a = -b$	8. _?_ property of equality

Written Exercises

Write the missing reasons. Assume that each variable represents any real number.

A **1.** *Prove:* If $a = b$, then $ca = cb$.
(Multiplication property of equality)

Proof:

1. $ca = ca$	1. _?_
2. $a = b$	2. Given
3. $ca = cb$	3. _?_

Write the missing reasons in Exercises 2–8. Assume that each variable represents any real number, except as noted.

2. *Prove:* If $a + c = b + c$, then $a = b$.

Proof:

1. $a + c = b + c$ — 1. Given
2. $(a + c) + (-c) = (b + c) + (-c)$ — 2. ?
3. $a + [c + (-c)] = b + [c + (-c)]$ — 3. ?
4. $a + 0 = b + 0$ — 4. ?
5. $a = b$ — 5. ?

3. *Prove:* If $ac = bc$ and $c \neq 0$, then $a = b$.

Proof:

1. $ac = bc$ — 1. Given
2. $(ac) \cdot \frac{1}{c} = (bc) \cdot \frac{1}{c}$ — 2. ?
3. $a\left(c \cdot \frac{1}{c}\right) = b\left(c \cdot \frac{1}{c}\right)$ — 3. ?
4. $a \cdot 1 = b \cdot 1$ — 4. ?
5. $a = b$ — 5. ?

4. *Prove:* If $b \neq 0$, then $\frac{1}{b}(ba) = a$ and $(ab)\frac{1}{b} = a$.

Proof:

1. $\frac{1}{b}(ba) = \left(\frac{1}{b} \cdot b\right)a$ — 1. ?
2. $= 1 \cdot a$ — 2. ?
3. $= a$ — 3. ?

From Step 3 prove that $(ab)\frac{1}{b} = a$.

4. $\frac{1}{b}(ba) = a$ — 4. (Step 3, above)
5. $(ba)\frac{1}{b} = a$ — 5. ?
6. $(ab)\frac{1}{b} = a$ — 6. ?

B

5. *Prove:* $-(-b) = b$

Proof:

1. $b + (-b) = 0$ — 1. ?
2. $(-b) + [-(-b)] = 0$ — 2. ?
3. $(-b) + [-(-b)] = b + (-b)$ — 3. ?
4. $[-(-b)] + (-b) = b + (-b)$ — 4. ?
5. $-(-b) = b$ — 5. Proved in Oral Exercise 2

6. *Prove:* $-(a + b) = (-a) + (-b)$
(Property of the opposite of a sum)

Proof: Since $-(a + b)$ is the unique additive inverse of $(a + b)$, we can prove that $-(a + b) = (-a) + (-b)$ by showing that the sum of $(a + b)$ and $[(-a) + (-b)]$ is 0.

(Proof continues on next page.)

1. $(a + b) + [(-a) + (-b)] = [(a + b) + (-a)] + (-b)$ 1. ___?___
2. $= [a + (-a) + b] + (-b)$ 2. ___?___
3. $= [0 + b] + (-b)$ 3. ___?___
4. $= b + (-b)$ 4. ___?___
5. $= 0$ 5. ___?___

7. *Prove:* $-(a - b) = b - a$

Proof:

1. $-(a - b) = -[a + (-b)]$	1. Definition of ___?___
2. $= (-a) + [-(-b)]$	2. Property of the opposite of a sum (proved in Exercise 6)
3. $= (-a) + b$	3. Proved in Exercise ___?___
4. $= b + (-a)$	4. ___?___
5. $= b - a$	5. ___?___

8. *Prove:* $-(-a - b) = a + b$

Proof:

1. $-(-a - b) = -[-a + (-b)]$	1. Definition of ___?___
2. $= -(-a) + [-(-b)]$	2. Property of the opposite of a sum (proved in Exercise 6)
3. $= a + b$	3. Proved in Exercise ___?___

Write proofs giving statements and reasons.

C **9.** *Prove:* If a and b are any real numbers, c is any nonzero real number, and $a = b$, then $\frac{a}{c} = \frac{b}{c}$. (*Hint:* Use Exercise 1.)

10. *Prove:* If a is any nonzero real number, then $\frac{a}{a} = 1$.

11. *Prove:* If a and b are nonzero real numbers, then $\frac{1}{\frac{a}{b}} = \frac{b}{a}$. (*Hint:* Show that $\frac{a}{b} \cdot \frac{b}{a} = 1$.)

Mixed Review Exercises

Simplify.

1. $12(12 - 7) \div 6 + 4$ **2.** $-4(-16 + 8)$ **3.** $-7x - 2x + 12x$

4. $-\frac{1}{2}(8 + 2a)$ **5.** $\frac{1}{2}(2b - 4) + 3$ **6.** $10 \div \frac{1}{2}$

Evaluate if $a = 5$, $b = 4$, $c = 3$, and $x = 6$.

7. $a(b + x)$ **8.** $3|x - a|$ **9.** $b - |x - a|$

10. $\frac{b + 2a}{|1 - c|x}$ **11.** $\frac{5a + x + 4}{b + c}$ **12.** $\frac{1}{2}(c - a) + x$

Self-Test 3

Vocabulary theorem (p. 130) proof (p. 130)

1. The length of a rectangle is 8 cm more than the width. A second rectangle is 5 cm wider and 6 cm longer than the first rectangle. The second rectangle has a perimeter of 242 cm. Find the dimensions of each rectangle. Make a sketch first. **Obj. 3-6, p. 121**

2. Jeremy had 34 nickels and quarters totaling $4.10. He had two less than twice as many nickels as quarters. How many of each did he have? **Obj. 3-7, p. 126**

3. Write the missing reasons. **Obj. 3-8, p. 130**

1.	$-a + (a + b) = (-a + a) + b$	1. ?
2.	$= 0 + b$	2. ?
3.	$= b$	3. ?

Check your answers with those at the back of the book.

Chapter Summary

1. The addition, subtraction, multiplication, and division properties of equality guarantee that:
   a. Adding the same real number to, or subtracting the same real number from, equal numbers gives equal results.
   b. Multiplying or dividing equal numbers by the same nonzero real number gives equal results.
2. Transforming an equation by substitution, by addition or subtraction, or by multiplication or division (not by zero) produces an equivalent equation. These transformations are used in solving equations.
3. Inverse operations are used in solving equations.
4. Equations can be used to solve word problems. Organizing the facts of a word problem in a chart is often helpful.
5. The following formulas are helpful in setting up charts to solve problems about cost, income, and value.

   Cost = number of items × price per item

   Income = hours worked × wage per hour

   Value = number of items × value per item
6. Theorems are proved by logically developing an argument to support them. In such a proof, each step is justified by a definition, property, or previously proven theorem.

Chapter Review

Write the letter of the correct answer.

1. Solve $20 = 5 + x$. 3-1
a. 25 **b.** -15 **c.** 15 **d.** -25

2. Solve $y - 17 = 19$.
a. -36 **b.** 2 **c.** 26 **d.** 36

3. Solve $\frac{1}{9}x = 5$. 3-2
a. $\frac{5}{9}$ **b.** $5\frac{1}{9}$ **c.** $4\frac{5}{9}$ **d.** 45

4. Solve $4n = -2$.
a. 6 **b.** 2 **c.** -2 **d.** $-\frac{1}{2}$

5. Solve $\frac{1}{3}x - 3 = -3$. 3-3
a. 0 **b.** -18 **c.** 18 **d.** 2

6. Solve $b - 3b = 24$.
a. -8 **b.** -12 **c.** 12 **d.** -6

7. Howard works an 8-hour day at his gas station. He spends twice as much time working on cars as he does waiting on customers. He takes $1\frac{1}{4}$ hours to eat lunch and balance his books. How many hours does he spend waiting on customers? 3-4
a. 2 h **b.** $2\frac{1}{4}$ h **c.** $1\frac{1}{4}$ h **d.** $4\frac{1}{2}$ h

8. Solve $2m = 1 - m$. 3-5
a. $\frac{1}{3}$ **b.** 2 **c.** 1 **d.** 3

9. Solve $3w - 13 = \frac{1}{4}(52 - 12w)$.
a. $4\frac{1}{3}$ **b.** -1 **c.** no solution **d.** identity

10. Arthur weighs 34 lb more than Lily. Their combined weight is 180 lb less than four times Lily's weight. How much does Arthur weigh? 3-6
a. 141 lb **b.** 151 lb **c.** 107 lb **d.** 127 lb

11. Nick worked 16 hours last week. He earned \$5 per hour at a local restaurant and \$5.50 per hour at a grocery store. If he earned a total of \$82, how many hours did he work at the grocery store? 3-7
a. 8 h **b.** 4 h **c.** 12 h **d.** 2 h

12. Which of the following properties should be given as the reason for the statement $a(bc) = a(cb)$? 3-8
a. Distributive property **b.** Associative property of multiplication
c. Property of opposites **d.** Commutative property of multiplication

Chapter Test

Solve.

1. $y + 25 = 10$ **2.** $73 = h - 13$ **3-1**

3. $c + 51 = 38$ **4.** $x - 38 = 12$

5. $\frac{1}{13}y = 65$ **6.** $-19v = -114$ **3-2**

7. $-112 = 16e$ **8.** $-\frac{x}{21} = 35$

9. $12y - 7 = 113$ **10.** $\frac{2}{3}x + 6 = 16$ **3-3**

11. $\frac{3x + 90}{5} = 0$ **12.** $-\frac{7}{8}(w - 16) = 70$

13. In the game of basketball you can score one point for a foul shot, two points for a regular shot and three points for an outside shot. Manuel scored 30 points by making eight foul shots and two outside shots. How many regular shots did he make? Use the five-step plan. **3-4**

Solve each equation. If the equation is an identity or if it has no solution, write *identity* or *no solution*.

14. $7(a - 6) = -3 + 6a$ **15.** $6(m - 1) = 6(m + 3)$ **3-5**

Solve. In Exercises 17 and 18, use a chart to help you.

16. Three times a number increased by 44 is the same as the opposite of the number. Find the number.

17. Sean weighs 10 lb more than twice Brad's weight. If Brad gains 10 lb, together they'll weigh 230 lb. How much does each weigh now? **3-6**

18. When Courtney collected her change she realized that she had five times as many dimes as quarters. Her dimes and quarters totaled $5.25. How many quarters did she have? **3-7**

19. Write the missing reasons to justify the multiplicative property of zero. **3-8**

1.	$0 = 0 + 0$	1. ?
2.	$a \cdot 0 = a(0 + 0)$	2. ?
3.	$a \cdot 0 = a \cdot 0 + a \cdot 0$	3. ?
4.	But $a \cdot 0 = a \cdot 0 + 0$	4. Identity property of addition
5.	$\therefore a \cdot 0 + a \cdot 0 = a \cdot 0 + 0$	5. ?
6.	$a \cdot 0 = 0$	6. ?
7.	$0 \cdot a = 0$	7. ?

Cumulative Review *(Chapters 1–3)*

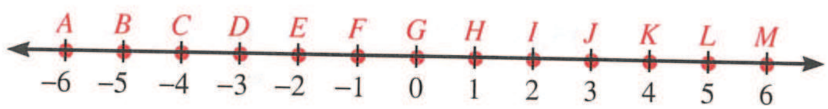
Simplify.

1. $45 - 2(16 - 6)$
2. $\frac{56 \div 7}{16}$
3. $48 \div (9 + 3)$
4. $|-21| - |-14|$
5. $31 - [35 \div 5]$
6. $48 \div 3 + 7(3)$
7. $-55 - (-42 + 7)$
8. $-22 + 31 + (-44) + 50$
9. $1\frac{1}{3} - 10\frac{1}{4} + 12\frac{2}{3}$
10. $2.4 + 5.1 - 6.3$
11. $9 + x - (5 - x) - 6$
12. $4(x + 3y) - 3(x - 4y)$
13. $-5(42)\left(-\frac{2}{5}\right)\left(-\frac{1}{3}\right)$
14. $-5(-a - b) + 5(a + b)$
15. $\frac{85xy}{5y}$; $y \neq 0$

Evaluate if $w = 2$, $x = -3$, $y = \frac{1}{2}$, **and** $z = 3$.

16. $w - |z - x|$
17. $\frac{4y - x}{w + z}$
18. $-5(x + w)$
19. $-w + 2x + 4y$

State the coordinate of the given point.



20. The point halfway between F and G.
21. The point halfway between B and C.

Solve. If the equation is an identity or has no solution, state that fact.

22. $|x| = 3$
23. $|y| = -2$
24. $x + 7 = 12$
25. $z + 1 = -3$
26. $y + 2 = 9$
27. $(x - 3) + 17 = 30$
28. $\frac{a}{3} = -14$
29. $9 = \frac{1}{2}q$
30. $2x + 6 = -2$
31. $\frac{1}{2}(2x + 4) = 2x$
32. $16 = \frac{3}{4}k + 1$
33. $5(z - 3) = 40$
34. $5y - 2 = 7y + 8$
35. $9(2 - b) = b$
36. $3(x - 4) = 6(x - 3)$

Solve.

37. A honeydew melon costs four times as much as a peach. Together they cost $1.50. How much does each cost?
38. Find three consecutive integers whose sum is 87.
39. Thirty-eight employees at High Tech Sales ride to work on the subway. This represents $\frac{2}{5}$ of the employees. How many employees are there?
40. Rory has 30 coins (all nickels and dimes). He has five times as many nickels as dimes. How much money does he have?

Maintaining Skills

Express each fraction as a mixed number.

Sample 1 $\frac{35}{13}$

Solution
$$\begin{array}{r} 2 \\ 13\,\overline{)35} \\ \underline{26} \\ 9 \end{array} \qquad \therefore \frac{35}{13} = 2\frac{9}{13}$$

1. $\frac{25}{12}$
2. $\frac{45}{6}$
3. $\frac{78}{15}$
4. $\frac{86}{20}$
5. $\frac{91}{12}$
6. $\frac{83}{7}$
7. $\frac{111}{12}$
8. $\frac{115}{13}$

Express each mixed number as a fraction.

Sample 2 $5\frac{2}{5}$

Solution $5\frac{2}{5} = 5 + \frac{2}{5} = \frac{25}{5} + \frac{2}{5} = \frac{27}{5}$

9. $4\frac{1}{6}$
10. $8\frac{3}{5}$
11. $2\frac{7}{9}$
12. $12\frac{3}{4}$
13. $3\frac{8}{13}$
14. $17\frac{1}{3}$
15. $9\frac{11}{12}$
16. $10\frac{7}{8}$

Perform the indicated operations. Express the answers in simplest form.

Sample 3 $8\frac{2}{3} + 7\frac{5}{6}$

Solution
$$8\frac{2}{3} + 7\frac{5}{6} = \frac{26}{3} + \frac{47}{6}$$
$$= \frac{52}{6} + \frac{47}{6}$$
$$= \frac{99}{6} = \frac{33}{2} = 16\frac{1}{2}$$

Sample 4 $3\frac{1}{3} \div 7\frac{1}{2}$

Solution
$$3\frac{1}{3} \div 7\frac{1}{2} = \frac{10}{3} \div \frac{15}{2}$$
$$= \frac{\overset{2}{\cancel{10}}}{3} \cdot \frac{2}{\underset{3}{\cancel{15}}} = \frac{4}{9}$$

17. $8\frac{7}{15} + 9\frac{11}{15}$
18. $5\frac{7}{9} - 6\frac{4}{9}$
19. $7\frac{1}{3} \cdot \left(-\frac{1}{7}\right)$
20. $4\frac{2}{5} \div 2\frac{1}{5}$
21. $10\frac{1}{5} \div \frac{2}{5}$
22. $20\frac{5}{13} + 8\frac{4}{5}$
23. $6\frac{5}{8} - 3\frac{2}{5}$
24. $6\frac{3}{5} \div 2\frac{3}{4}$
25. $12\frac{9}{10} - 8\frac{3}{5}$
26. $-9\frac{1}{2} + \left(-13\frac{5}{9}\right)$
27. $11\frac{4}{7} + 10\frac{1}{5}$
28. $8\frac{5}{6} \div 6\frac{5}{8}$
29. $4\frac{10}{11} - 7\frac{1}{2}$
30. $12\frac{3}{20} \cdot 1\frac{5}{9}$
31. $9\frac{5}{9} - 8\frac{1}{4}$
32. $3\frac{9}{10} \cdot 7\frac{1}{13}$
33. $-5\frac{2}{3} \div 3\frac{1}{2}$
34. $5\frac{5}{9} \cdot 4\frac{1}{2}$
35. $17\frac{3}{4} + 8\frac{1}{3}$
36. $10\frac{5}{6} \cdot 3\frac{4}{5}$

Mixed Problem Solving

Solve each problem that has a solution. If a problem has no solution, explain why.

A

1. The sum of twice a number and −6 is 9 more than the opposite of the number. Find the number.
2. Roger spent \$22 on a baseball mitt and softball. If the mitt cost \$2 less than 5 times the cost of the softball, find the cost of each.
3. I drove 450 km in 6 h. Find my rate of travel.
4. The Longs' checking account was overdrawn by \$35.87. They deposited \$580 in the account. Then they wrote checks for \$25 and \$254.09. Find their new balance.
5. Alice bought 12 apples and oranges for \$2.51. If an apple costs 25¢ and an orange costs 18¢, how many of each did she buy?
6. A rectangle has a perimeter of 48 cm. If the width and the length are consecutive odd integers, find the dimensions of the rectangle.
7. The usual July temperature in Windsor, Ontario, 22° C, is 27° above the usual January temperature. Find the usual January temperature.
8. When 7 is decreased by a number, the result is 10. Find the number.
9. Find three consecutive integers such that three times the smallest is equal to the middle number increased by the greatest number.
10. What is the difference between the boiling point of mercury, 357° C, and the melting point, −39° C?
11. Ruwa has \$125 in \$5 bills and \$10 bills. If he has four more \$5 bills than \$10 bills, how many of each does he have?

B

12. A store manager bought c calculators for \$8 each. All but four were sold for \$10 each. The remaining four calculators were not sold. Find the store's profit, in simplified form, in terms of c.
13. At a city zoo, about \$45 of every \$100 spent is used for animal care and supplies. One year \$216,000 was spent on these uses. Find the total zoo budget that year.
14. Denise did $\frac{7}{8}$ of the problems on a quiz correctly and five incorrectly. She did all the problems. How many were there?
15. On Saturday Kim worked three hours more than Ann did. Together, they worked one hour less than three times the hours Ann worked. How many hours did Kim work?
16. A bank contains 44 coins (nickels, dimes, and quarters). There are twice as many dimes as nickels and 8 fewer nickels than quarters. How much money is in the bank?
17. Sara has twice as much money as Miguel. If she had \$6 more, she would have $\frac{4}{3}$ as much money as he has. How much money does each have now?