Math 112 - Quiz 6 Name Key

September 14, 2018

No calculators. Show your work.

1. Let $f(x) = x^3 + 2x^2$ and $g(x) = 3x^2 - 1$. Find the function $\frac{f}{g}$ and state its domain. (2 pts.)

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x^3+2x^2}{3x^2-1} \quad \text{where } 3x^2-1 \neq 0$$

$$3x^2-1 = 0 \Rightarrow x^2 = \frac{1}{3} \Rightarrow x = \pm\sqrt{\frac{1}{3}} = \pm\frac{1}{\sqrt{3}}$$

$$\mathrm{Dom}\left(\frac{f}{g}\right) = \left\{x \in \mathbb{R} \mid x \neq \pm\frac{1}{\sqrt{3}}\right\}$$

2. Let $f(x) = x + \frac{1}{x}$ and $g(x) = \frac{x+1}{x+2}$. Find the composite function $g \circ f$ and state its domain. (3 pts.)

$$(g\circ f)(x) = g(f(x)) = \frac{f(x)+1}{f(x)+2} = \frac{x+\frac{1}{x}+1}{x+\frac{1}{x}+2}.$$

Multiplying numerator and denominator by x, we get

$$(g\circ f)(x) = \frac{x^2+x+1}{x^2+2x+1} = \frac{x^2+x+1}{(x+1)^2}$$

Note that $g(f(x))$ is not defined at $x=0$ b/c $f(x)$ is not defined there. Also we see from the encircled that $g(f(x))$ is also not defined at $x=-1$. Hence,

$$\mathrm{dom}(g\circ f) = \{x \in \mathbb{R} \mid x \neq -1, 0\}$$

/5

3. Write the function $F(x) = \dfrac{\sqrt[3]{x}}{1+\sqrt[3]{x}}$ in the form $f \circ g$. (2 pts.)

Let $f(x) = \dfrac{x}{1+x}$ and $g(x) = \sqrt[3]{x}$. Then

$$(f \circ g)(x) = f(g(x)) = \frac{g(x)}{1+g(x)} = \frac{\sqrt[3]{x}}{1+\sqrt[3]{x}}.$$

4. A stone is dropped into a lake, creating a circular ripple that travels outward at a speed of 20 inches per second. Express the radius of the circle as a function of the time t in seconds. Let A denote the area of the circle. Find $A \circ r$. (3 pts.)

Let r denote the radius (in inches) of a circular ripple at time t seconds. Since its speed is a constant $20 \frac{in}{s}$,

$$r = \left(20 \tfrac{in}{s}\right)(t\ s) = 20t \text{ in}.$$

Since the area A of a circle of radius r is

$$A = \pi r^2,$$

the composite function $A \circ r$ is

$$(A \circ r)(t) = A(r(t)) = \pi [r(t)]^2 = \pi (20t)^2$$

$$\therefore (A \circ r)(t) = 400\pi t^2 \text{ in}^2$$

Extra Credit. (1 pt.)
Determine whether the function $h(x) = x|x|$ is even, odd, or neither.

Replace x with $-x$:

$$h(-x) = -x|-x| = -x|x| \quad \text{since } |-x| = |x|$$

Thus, $h(-x) = -h(x)$. So h is an odd function.