

## 1-3 Equations

**Objective:** To find solution sets of equations over a given domain.

### Vocabulary

**Equation** An equation is formed by placing an equals sign between two numerical or variable expressions. Examples:  $2 + 3 = 5$ ,  $x - 1 = 7$

**Open sentence** A sentence containing variables.

**Domain of a variable** The given set of numbers a variable may represent.

**Solution, or root, of an equation** A value of a variable that turns an open sentence into a true statement. For example, 8 is the solution of the equation  $x - 1 = 7$ .

**Solution set of an equation** The set of all the solutions of an equation.

### Symbol

$\in$  (is an element of, or belongs to)

**Example 1** The domain of  $x$  is  $\{0, 1, 2\}$ .  
Is the equation  $3x - 1 = 5$  true when  $x = 0$ ? when  $x = 1$ ? when  $x = 2$ ?

**Solution** Replace  $x$  in turn by 0, 1, and 2.

$x$	$3x - 1 = 5$	
0	$3 \cdot 0 - 1 = 5$	False
1	$3 \cdot 1 - 1 = 5$	False
2	$3 \cdot 2 - 1 = 5$	True

**Example 2** Read: a.  $y \in \{1, 2, 3\}$  b.  $x \in \{0, 2, 4, 6\}$

**Solution** a.  $y$  belongs to the set whose members are 1, 2, and 3.  
b.  $x$  belongs to the set whose members are 0, 2, 4, and 6.

**Example 3** Solve  $y(3 - y) = 2$  if  $y \in \{0, 1, 2, 3\}$ .

**Solution** Replace  $y$  in turn with 0, 1, 2, and 3.

$y$	$y(3 - y) = 2$	
0	$0(3 - 0) = 2$	False
1	$1(3 - 1) = 2$	True
2	$2(3 - 2) = 2$	True
3	$3(3 - 3) = 2$	False

The solutions are 1 and 2.

The solution set is  $\{1, 2\}$ .

**1–3 Equations** (continued)Solve each equation if  $x \in \{0, 1, 2, 3, 4, 5\}$ .

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|---------------------|----------------------|------------------------|
| 1. $x + 3 = 7$      | 2. $5 + x = 9$       | 3. $x - 3 = 2$         |
| 4. $x - 1 = 3$      | 5. $6 - x = 1$       | 6. $5 - x = 2$         |
| 7. $x + 3 = 3$      | 8. $x + 2 = 5$       | 9. $2x = 8$            |
| 10. $3x = 12$       | 11. $4x = 0$         | 12. $5x = 25$          |
| 13. $x \div 2 = 1$  | 14. $x \div 1 = 3$   | 15. $\frac{1}{2}x = 1$ |
| 16. $4x = 16$       | 17. $x \cdot x = 4$  | 18. $5x = 5$           |
| 19. $x \cdot x = 9$ | 20. $x \cdot x = 16$ | 21. $3x + 7 = 19$      |
| 22. $5x - 2 = 13$   | 23. $x(5 - x) = 6$   | 24. $x(4 - x) = 3$     |

**Example 4** Solve over the domain  $\{2, 4, 6\}$ .  
Three more than twice a number is 11. What is the number?

**Solution** Use mental math to see which members of the given domain are solutions.

Number	Three more than twice a number is 11.	
2	Three more than twice 2 is 11.	False
4	Three more than twice 4 is 11.	True
6	Three more than twice 6 is 11.	False

The number is 4.

Solve each problem over the domain  $\{2, 3, 4, 5\}$ .

25. Eleven more than a number is 15. What is the number?
26. Four times a number is 16. What is the number?
27. A number divided by one is 5. What is the number?
28. Two less than a number is 3. What is the number?
29. One less than twice a number is 9. What is the number?
30. One more than twice a number is 7. What is the number?

**Mixed Review Exercises**

Simplify.

- |                             |                           |                             |
|-----------------------------|---------------------------|-----------------------------|
| 1. $9 \cdot 8 + 9 \cdot 12$ | 2. $8 + (12 \div 2)$      | 3. $(16 - 7) \div 3$        |
| 4. $(2 + 3 \cdot 4) \div 7$ | 5. $15 - 9 \div 3 \div 3$ | 6. $35 \div 7 \div (3 + 2)$ |

Evaluate if  $a = 2$ ,  $x = 3$ ,  $y = 5$ , and  $z = 6$ .

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|----------------|------------------------|-----------------------|
| 7. $2x + 3y$   | 8. $8 \cdot (z - x)$   | 9. $3(ax + 2)$        |
| 10. $3xz + 2y$ | 11. $axz \div (y + 1)$ | 12. $4a \div (x + 1)$ |